

# LES of Flow Past Cylinders and Airfoils

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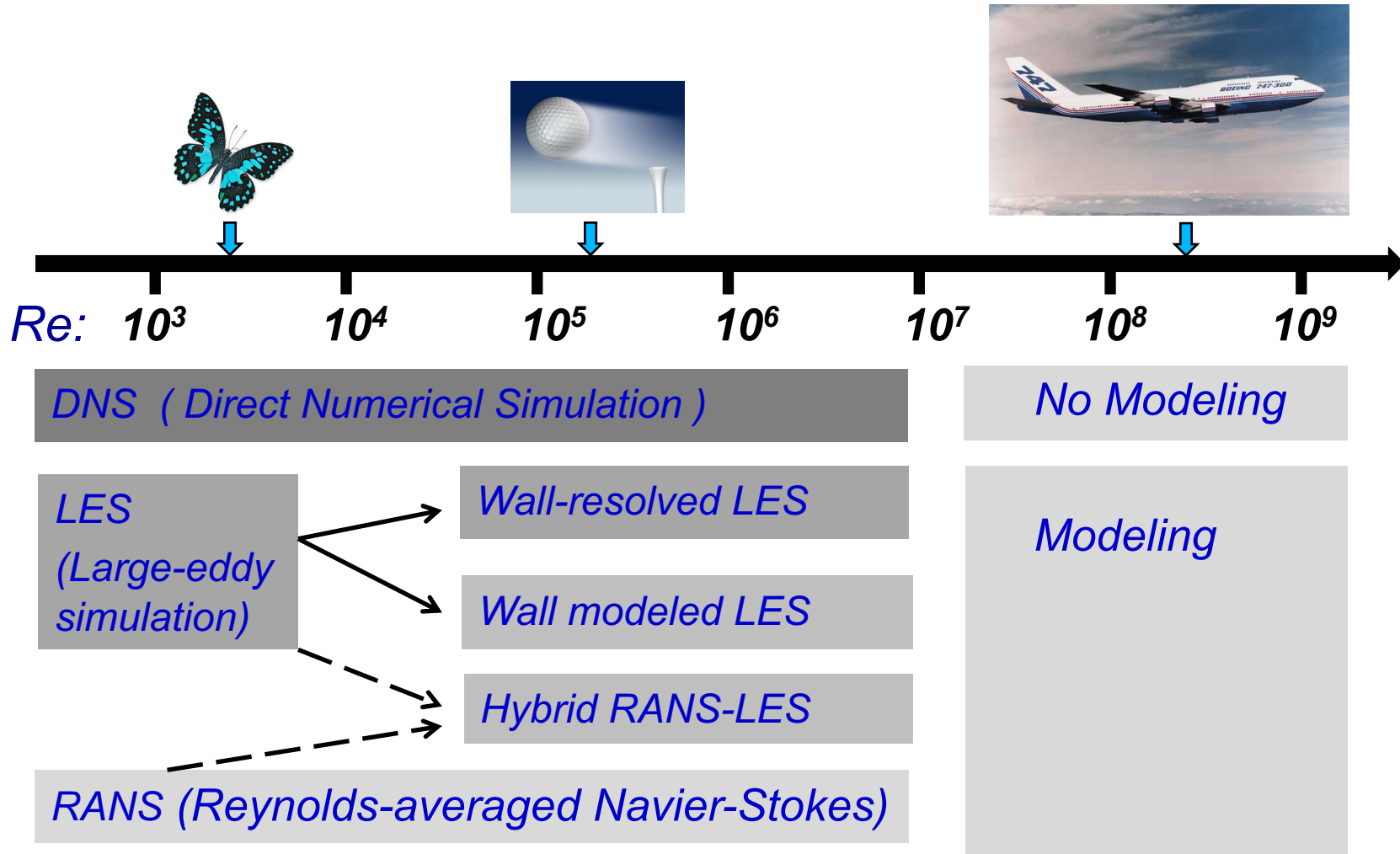
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California Institute of Technology

# Outline

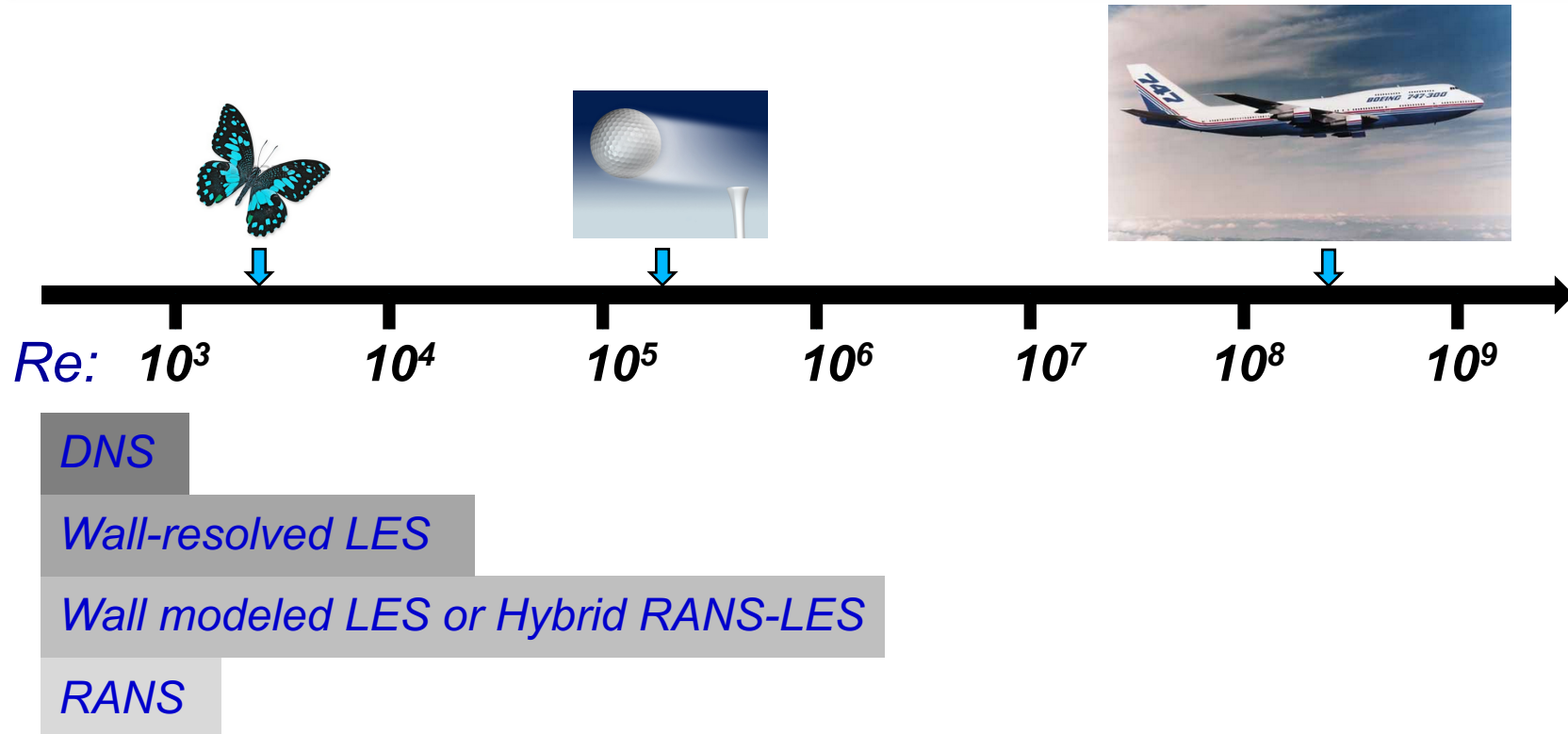
- Introduction
- Filtered NS Equations and SGS Model
- Wall-resolved large-eddy simulations (LES)
  - *LES of flow past a cylinder*
    - *Smooth*
    - *Grooved*
    - *Rotating*
- Wall modeled LES
  - *Virtual wall boundary conditions*
- WMLES of flow past airfoils
- Conclusion



# Fluid Mechanics: Reynolds number ( $Re$ )

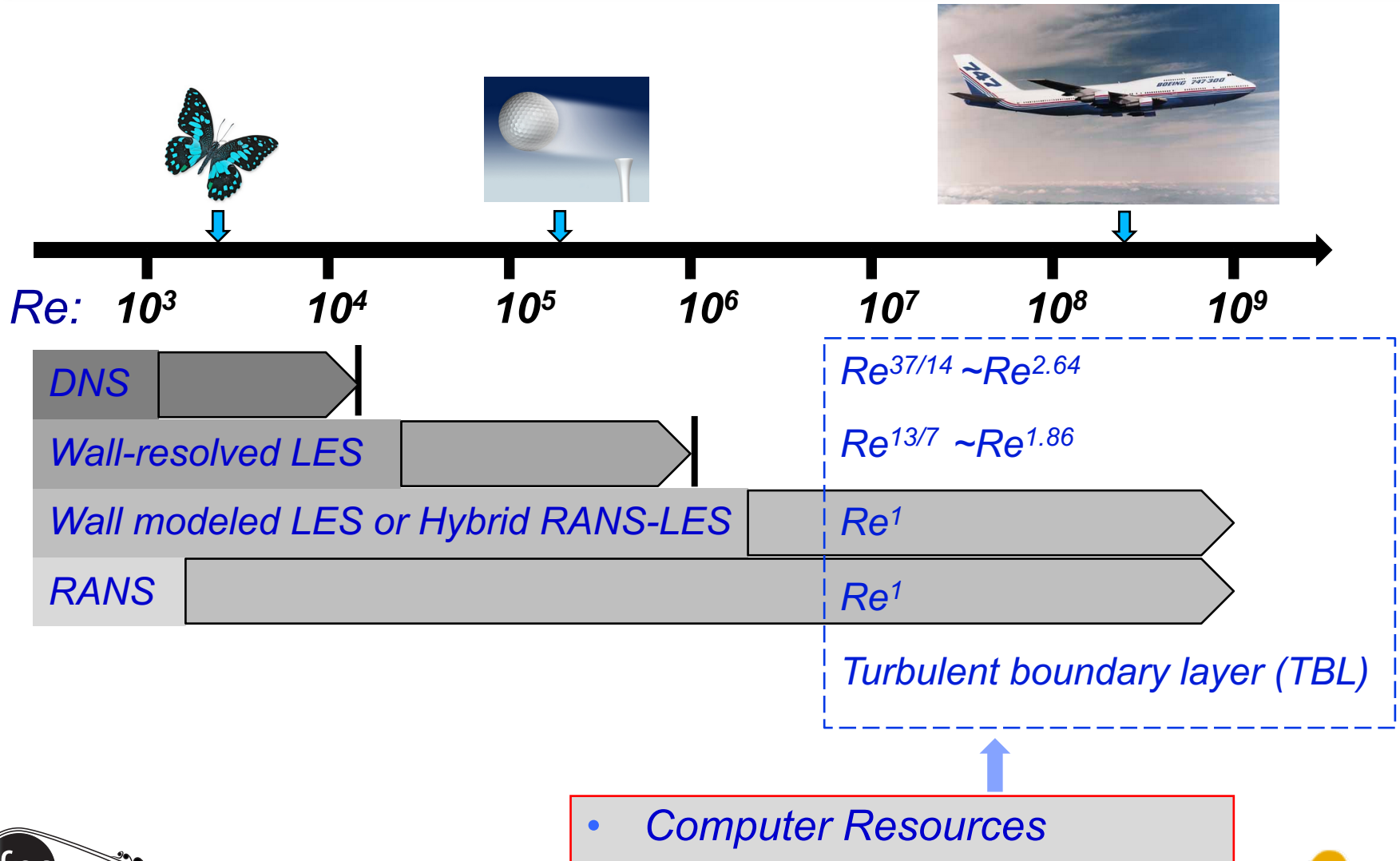


# Fluid Mechanics: Reynolds number ( $Re$ )

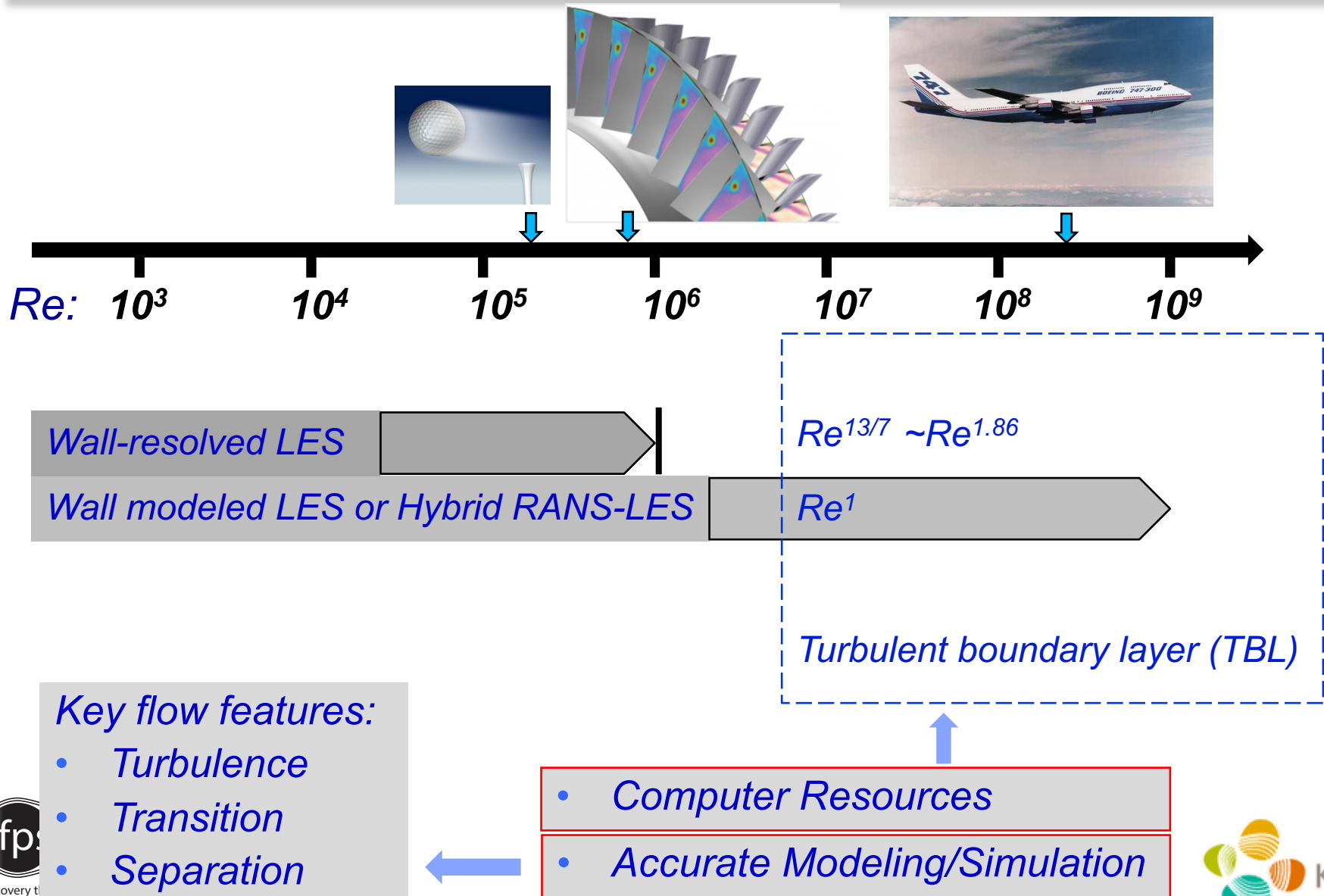


- *Computer Resources*

# Fluid Mechanics: Reynolds number ( $Re$ )

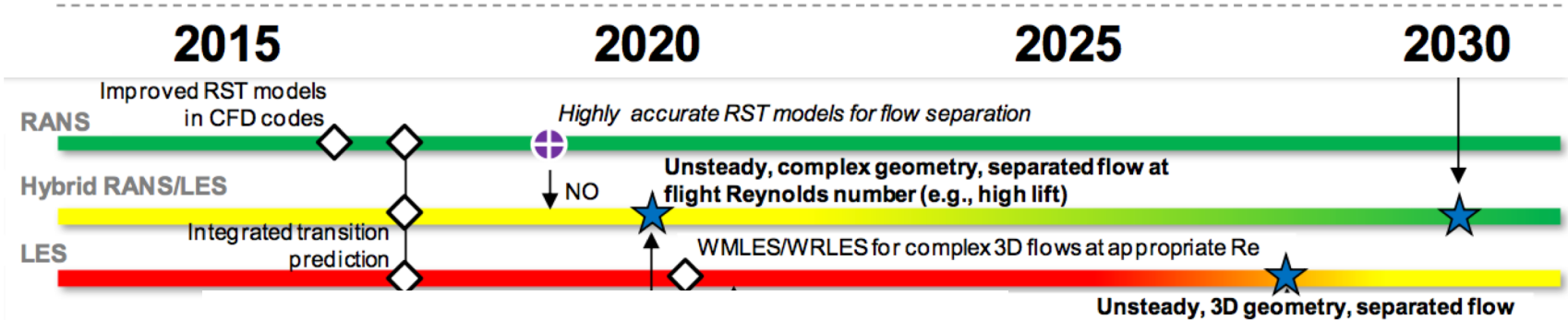


# Fluid Mechanics: Reynolds number ( $Re$ )



## NASA: Critical challenges in CFD ---- Separation

- 



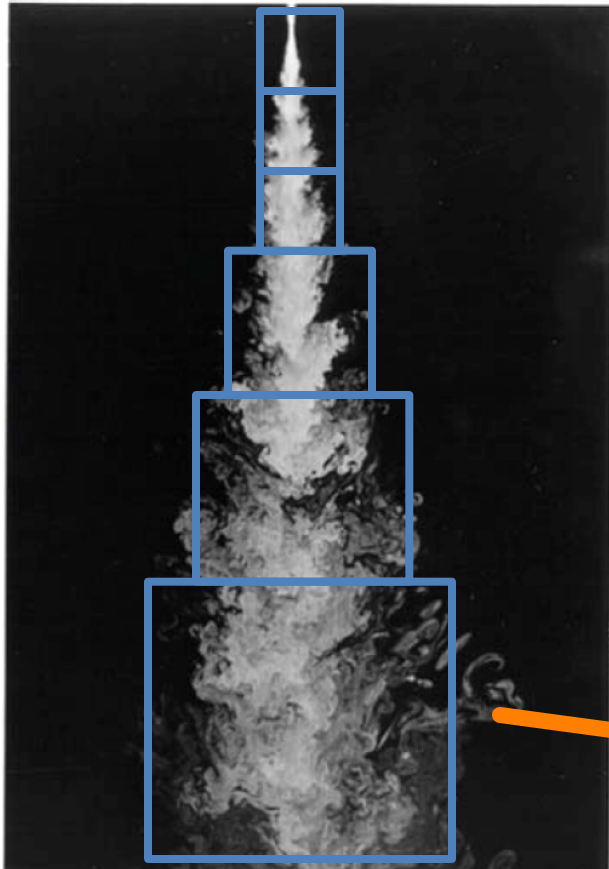
# Direct Numerical Simulation (DNS) Large-Eddy Simulation (LES)



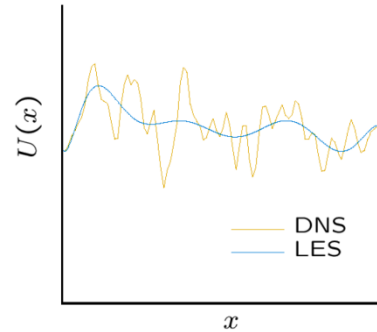
*Re = 10k (Dimotakis et al.  
1983)*

# Direct Numerical Simulation (DNS)

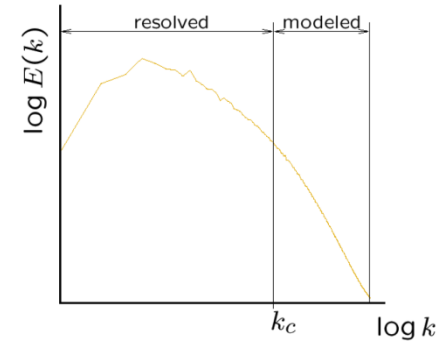
## Large-Eddy Simulation (LES)



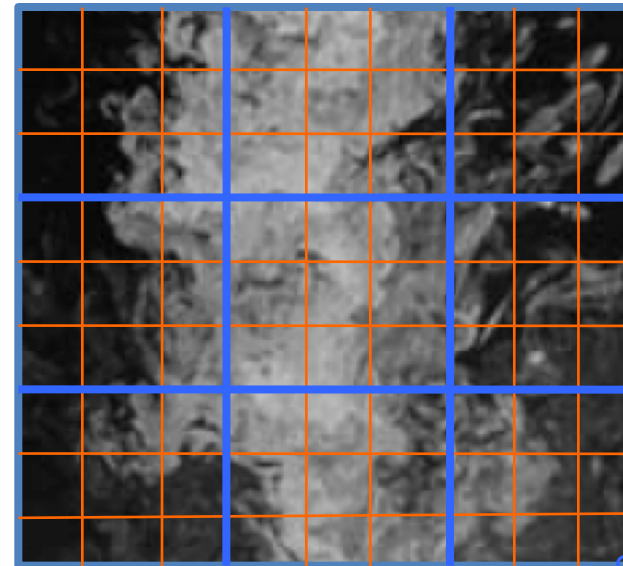
*Re = 10k (Dimotakis et al.  
1983)*



physical space: fine-scale fluctuations not resolved, their influence is modeled.



spectral space: resolved range,  $k < k_c$  (cutoff wavenumber  $k_c$ ), subgrid range  $k > k_c$ .



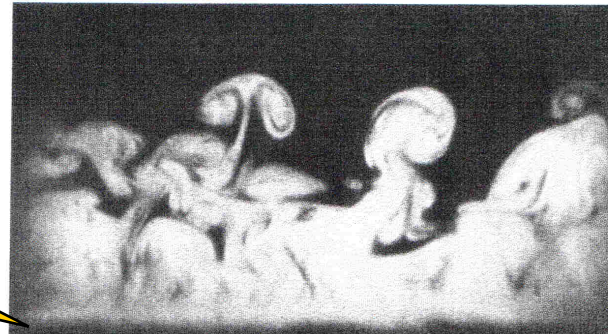
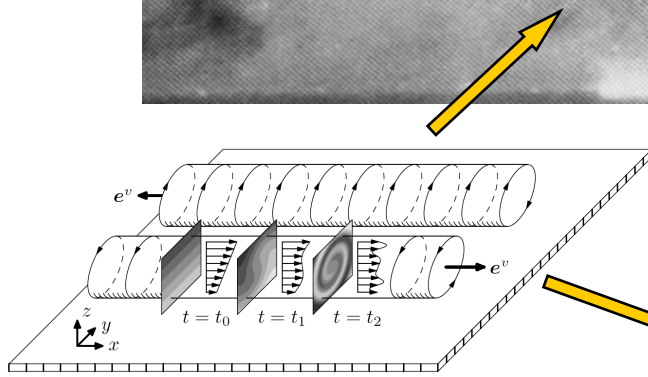
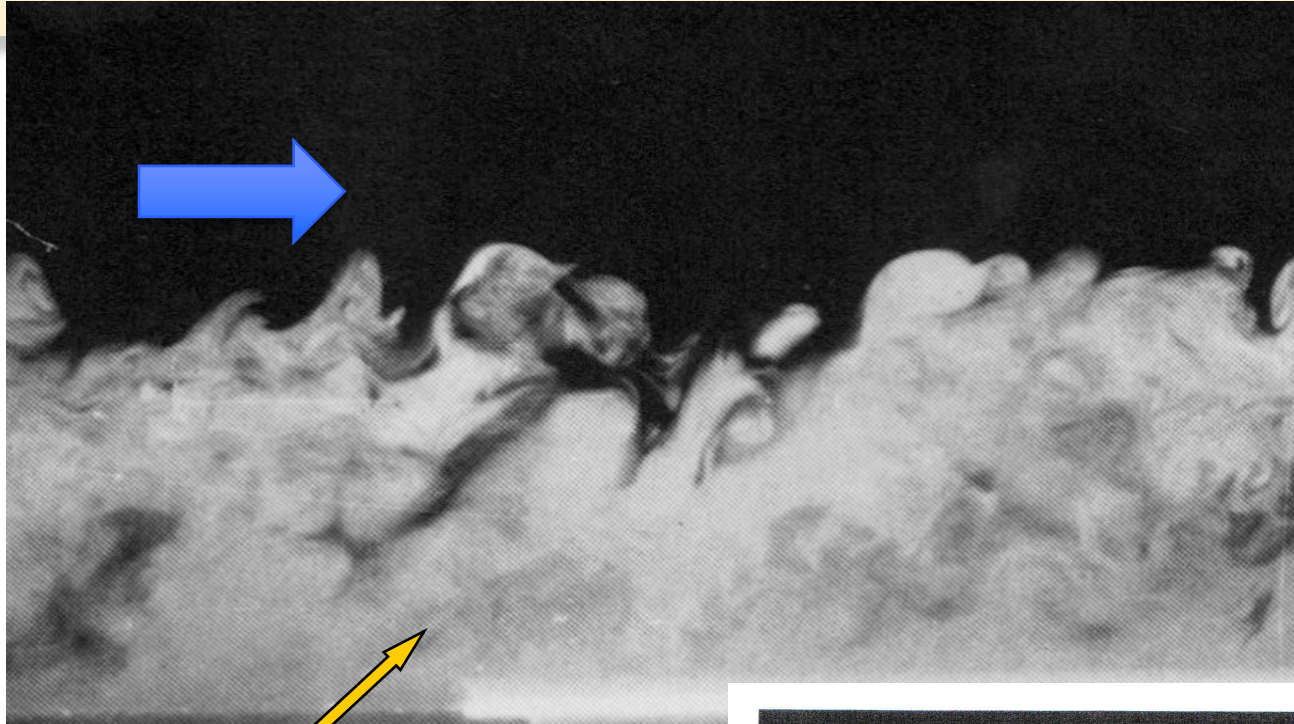
**DNS**

**LES**



# LES for wall-bounded flows

(Falco 1977)



Head & Bandyopandhyay (1981)



# Filtered Navier-Stokes Equations

- Apply filtering operation to incompressible Navier-Stokes equations

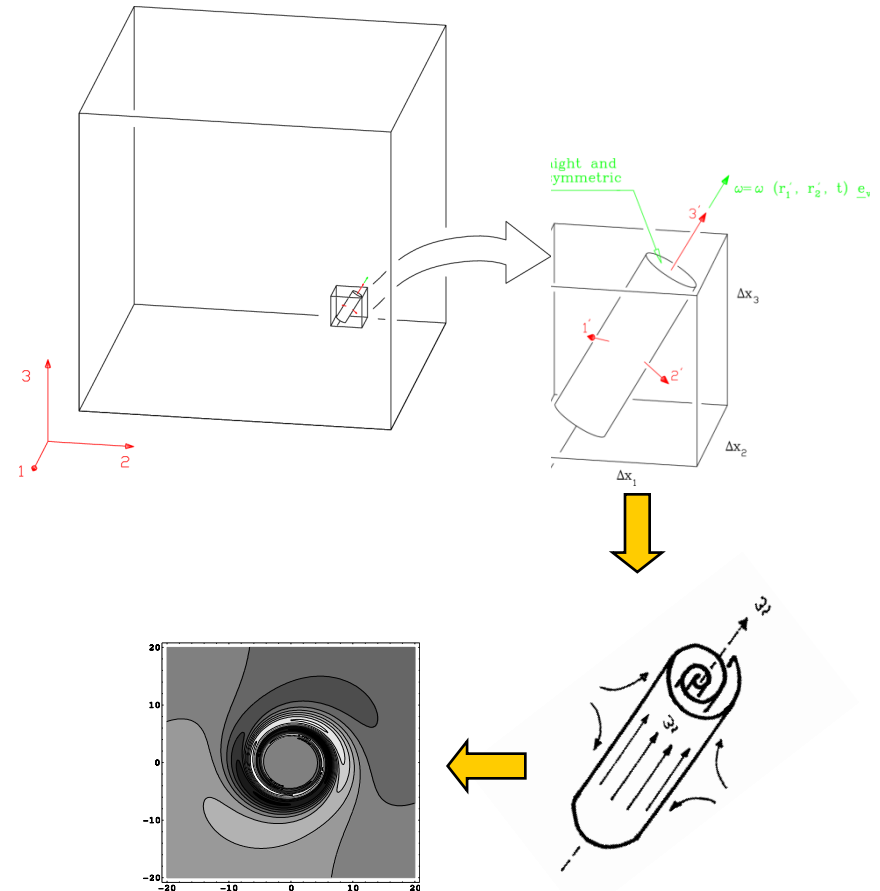
$$\begin{aligned}\frac{\partial \tilde{u}_i}{\partial x_i} &= 0 \\ \frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\widetilde{u_i u_j}) &= -\frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_i^2} \\ \frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\tilde{u}_i \tilde{u}_j) &= -\frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial}{\partial x_i} \underbrace{(\widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j)}_{T_{ij}} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_i^2} \\ \widetilde{u_i u_j}(\mathbf{x}, t) &\equiv \int_{-\infty}^{\infty} \mathcal{G}(\mathbf{x}' - \mathbf{x}) u_i(\mathbf{x}') u_j(\mathbf{x}') d\mathbf{x}' \neq \tilde{u}_i \tilde{u}_j\end{aligned}$$

- $T_{ij}$  “unresolved stresses” must be modeled: this is the “closure problem”
- This equation set is NOT closed
- Filtering process on NS equations is strictly formal: no particular filter is actually needed

# Explicit SGS model: stretched-vortex model

- Structure-based approach
- Subgrid motion represented by nearly axisymmetric vortex tube within each cell
- Local solution of NS equations for stretched-spiral vortex
  - *Lundgren (1982), Pullin & Lundgren (2001)*
- Subgrid stress:

$$T_{ij} = (\delta_{ij} - e_i^v e_j^v) K,$$



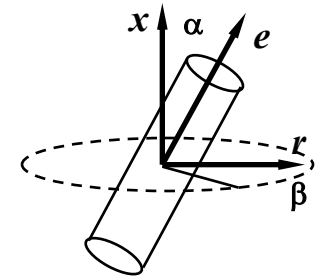
$$K = \int_{k_c}^{\infty} E(k) dk = \frac{1}{2} \mathcal{K}'_0 \Gamma[-1/3, \kappa_c^2], \mathcal{K}'_0 = \mathcal{K}_0 \epsilon^{2/3} \lambda_v^{2/3}$$

# Model parameters

- Subgrid energy spectrum (Lundgren, 1982)

$$E(k) = \mathcal{K}_0 \epsilon^{2/3} k^{-5/3} \exp[-2k^2 \nu / (3|\tilde{a}|)]$$

$$\tilde{a} = \tilde{S}_{ij} e_i^v e_j^v, \quad \tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)$$

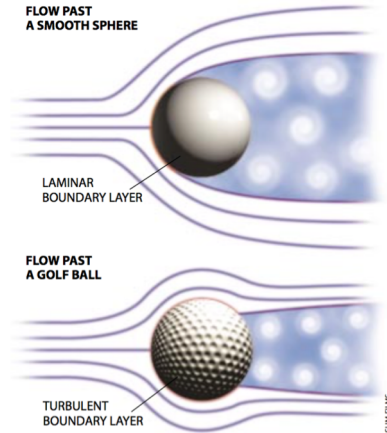
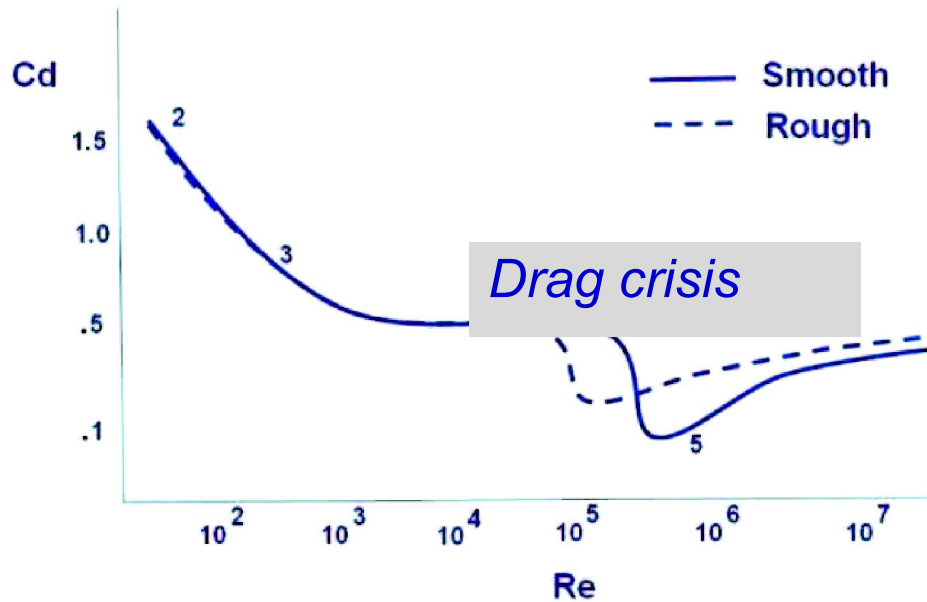


- Parameters obtained from resolved-scale, second order velocity structure-functions (Lesieur et al)

$$\mathcal{K}_0 \epsilon^{2/3} = \frac{\overline{\mathcal{F}_2(\Delta)}}{\Delta^{2/3} A}, \quad A = 4 \int_0^\pi s^{-5/3} (1 - s^{-1} \sin s) ds \approx 1.90695$$

$$\overline{\mathcal{F}_2(\Delta)} = \frac{1}{6} \sum_{j=1}^3 \left( \delta u_1^{+2} + \delta u_2^{+2} + \delta u_3^{+2} + \delta u_1^{-2} + \delta u_2^{-2} + \delta u_3^{-2} \right)_j$$

# Motivation: Drag Crisis



"Tackling turbulence using Supercomputers" –Kim & Moin 1997

## Golf ball trajectory

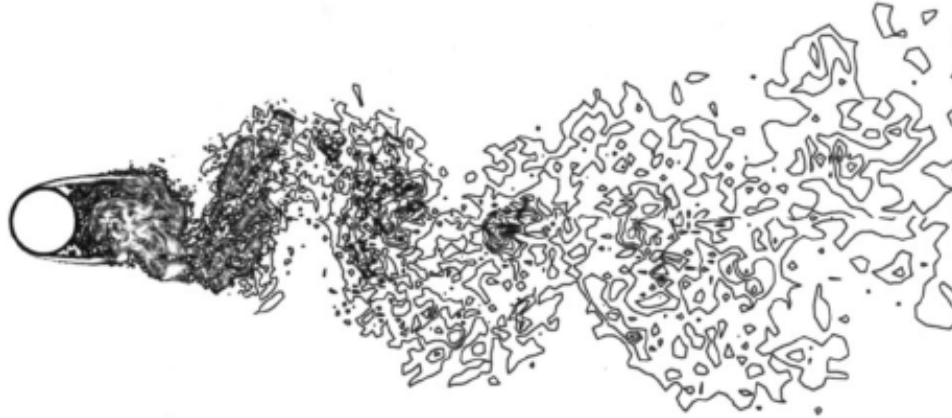
(<https://www.golf-simulators.com/physics.htm>)



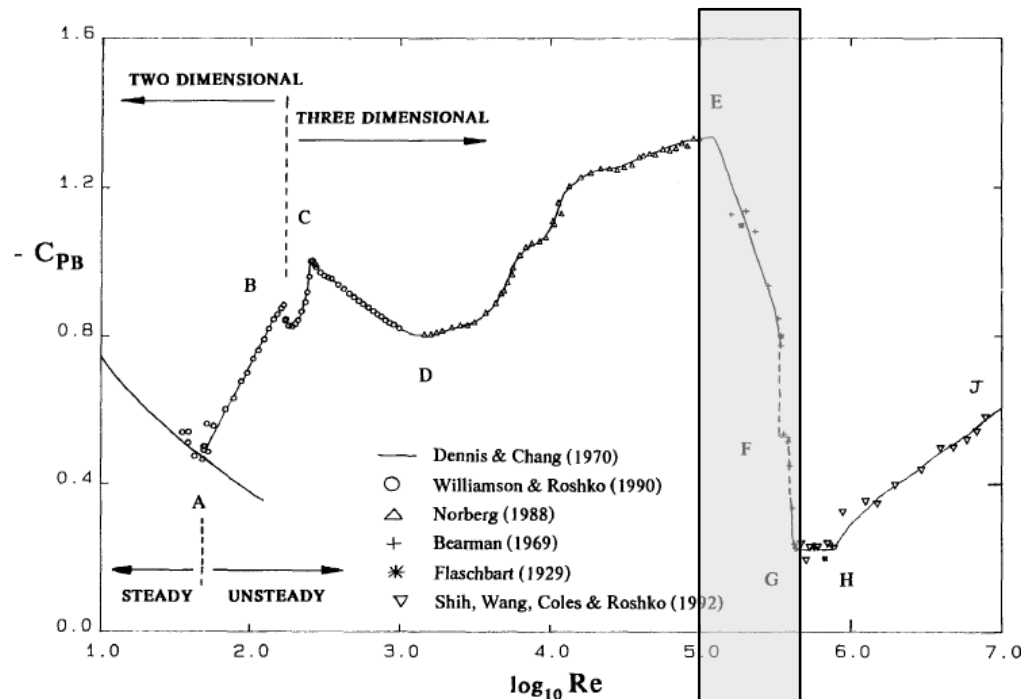
# Flow past a cylinder

- SC: Smooth Cylinder
- GC: Grooved Cylinder
- RC: Rotating Cylinder

# SC: Background of Flow past a cylinder



Kravchenko & Moin, PoF, 2000



## Drag crisis

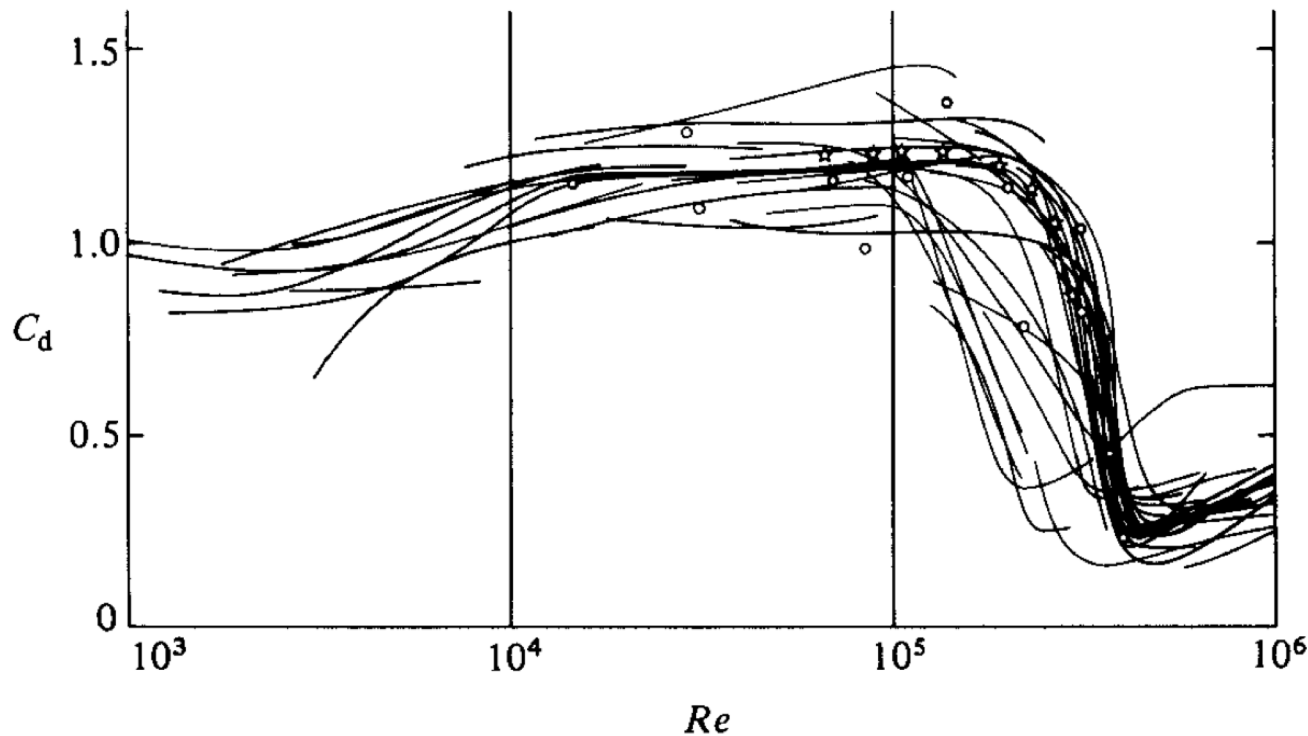
### Conventional explanation

- *Turbulence location*
  - Wake in subcritical
  - BL for supercritical
- *Any other explanation?*

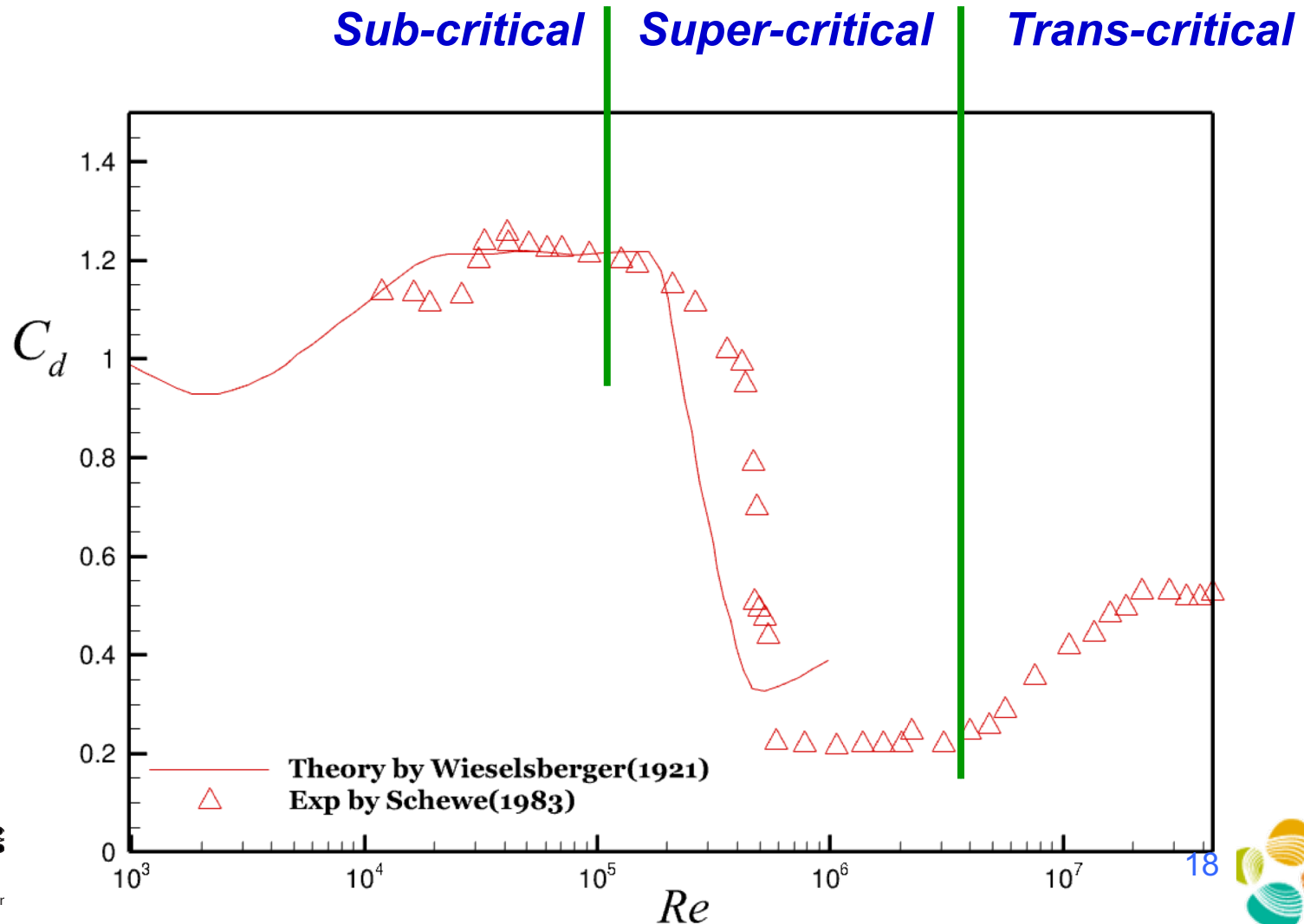
Roshko, JWEIA, 1993

# SC: Drag Coefficient

- Drag coefficient results from experiments do not agree with each other
- Figure from Cantwell & Coles (1983, JFM): exp. Data from literature

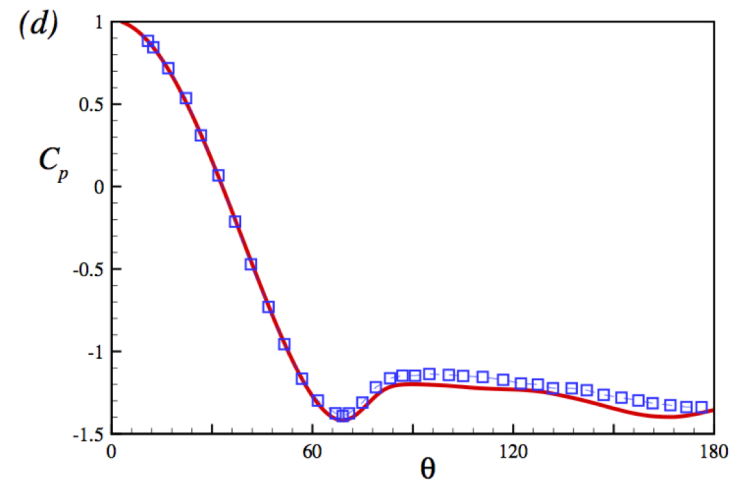
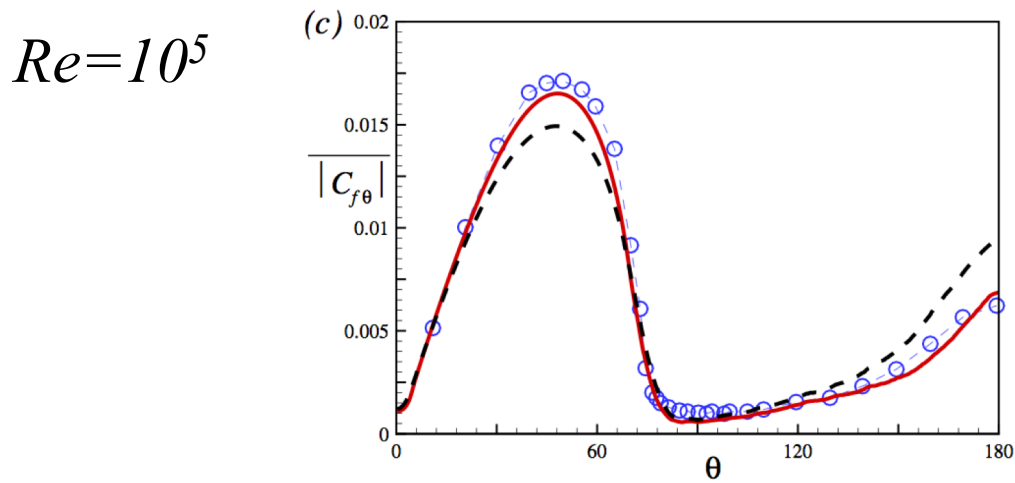
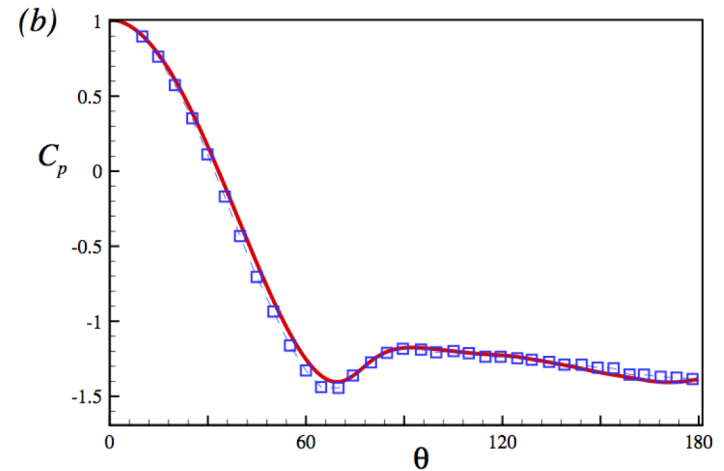
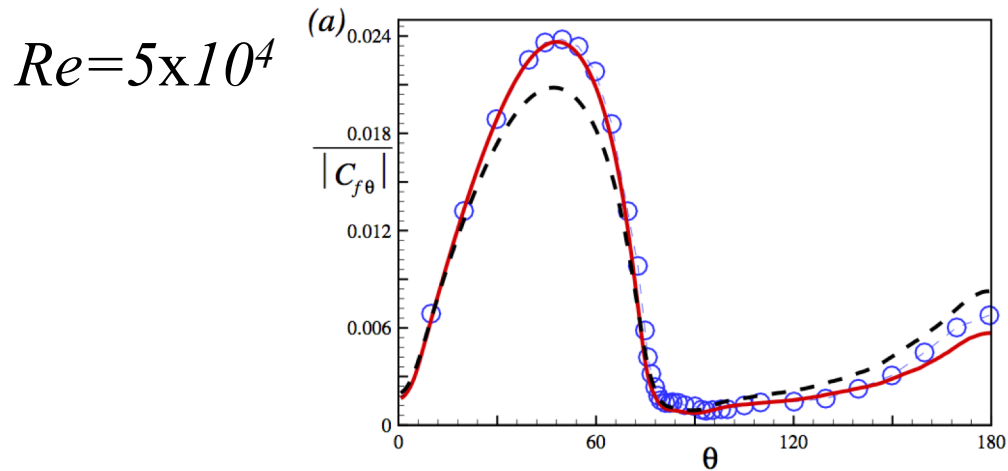


# SC: Drag Coefficient



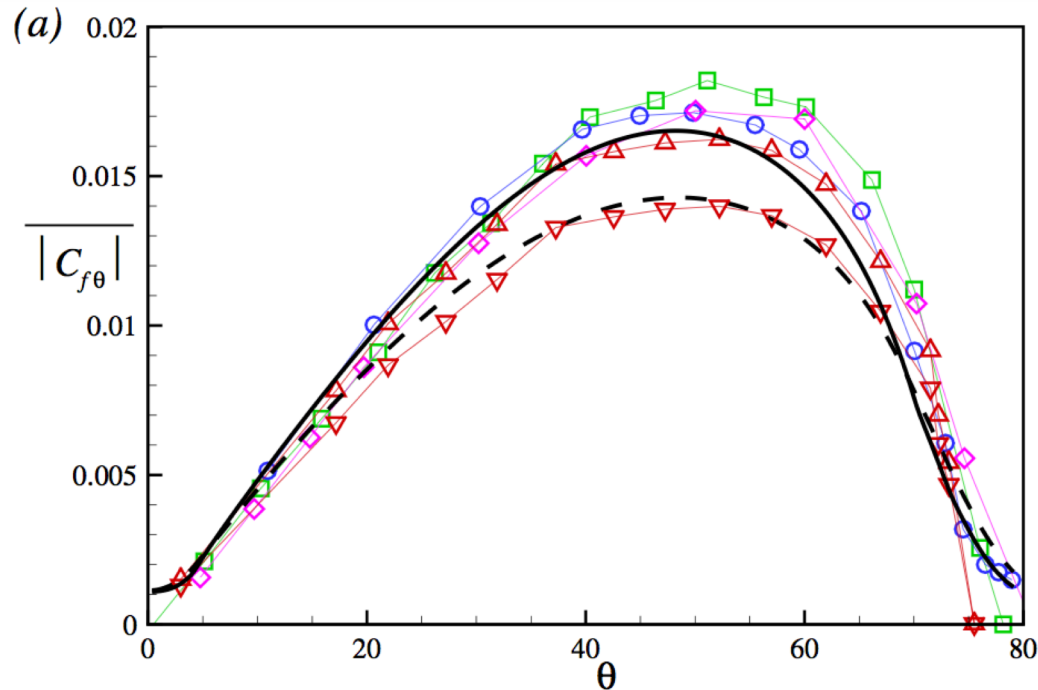


# SC: Sub-critical cases



- *Passively using wall-model ODE gives best results when mesh is not fine enough*

# SC: $Re=10^5$

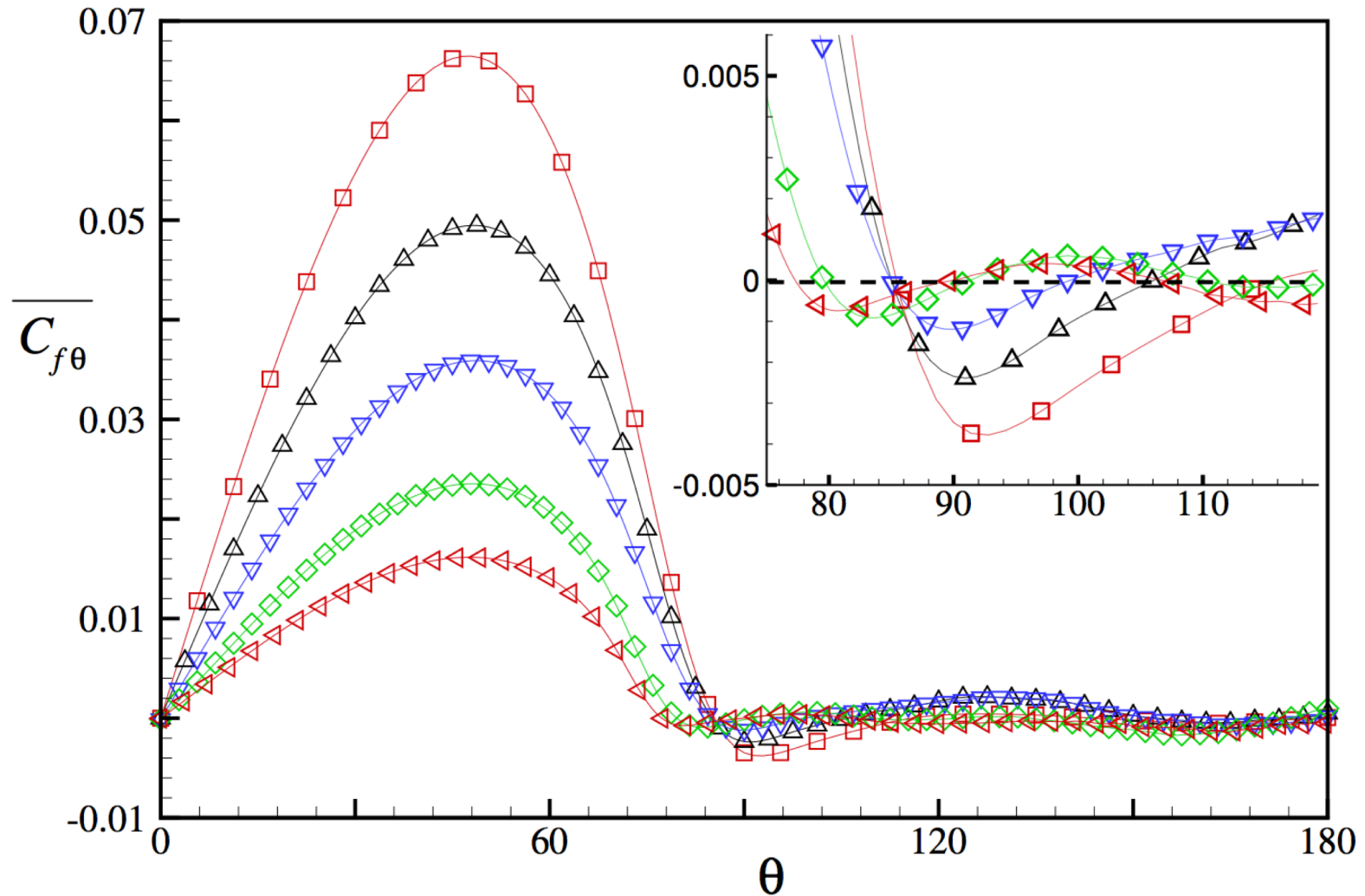


- □ Exp by Achenbach (1968)
- ○ Exp by Son & Haraty (1969)
- ◇ Exp by Giedt (1951)
- △ Theory by Thom (1928)
- using ODE
- ▽ Exp by Thom (1928)
- - - velocity derivative

} Diagnosed via momentum conservation law

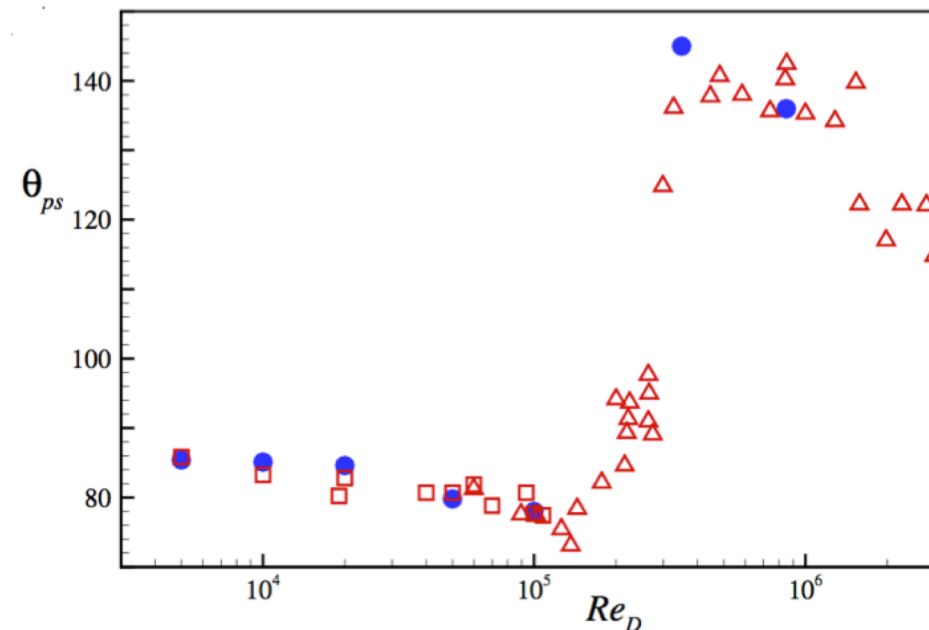
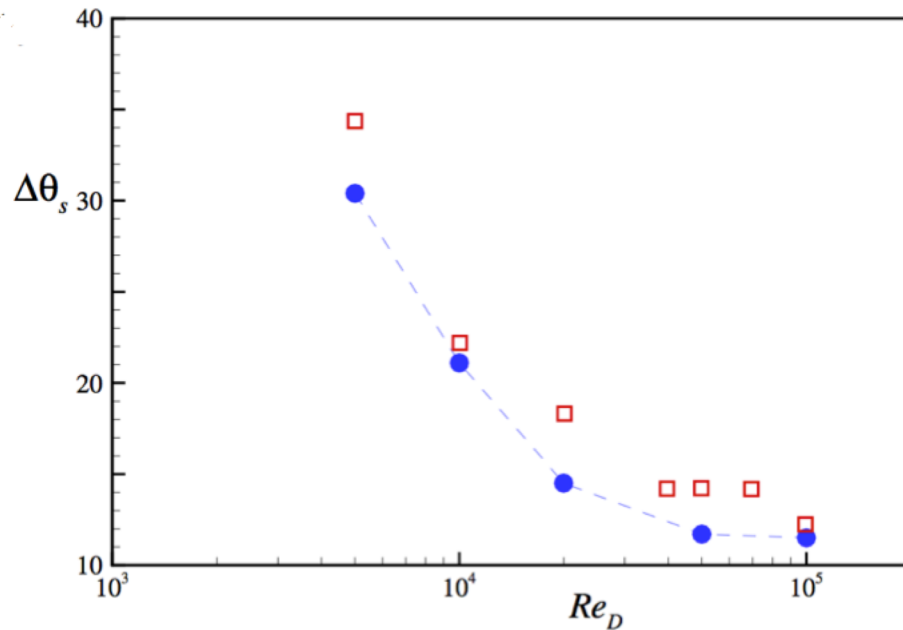
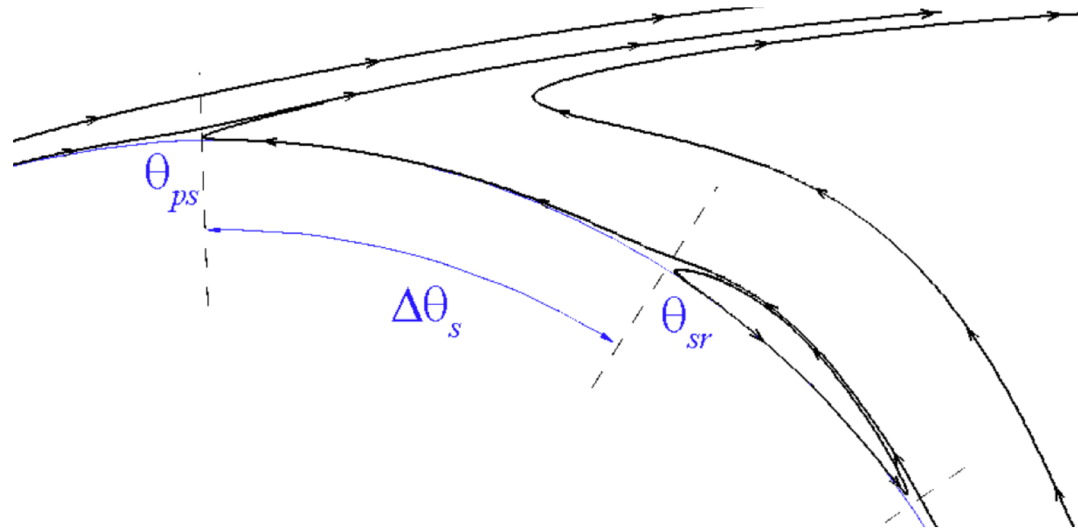
} Diagnosed via velocity component

# SC: Separation: sub-critical cases

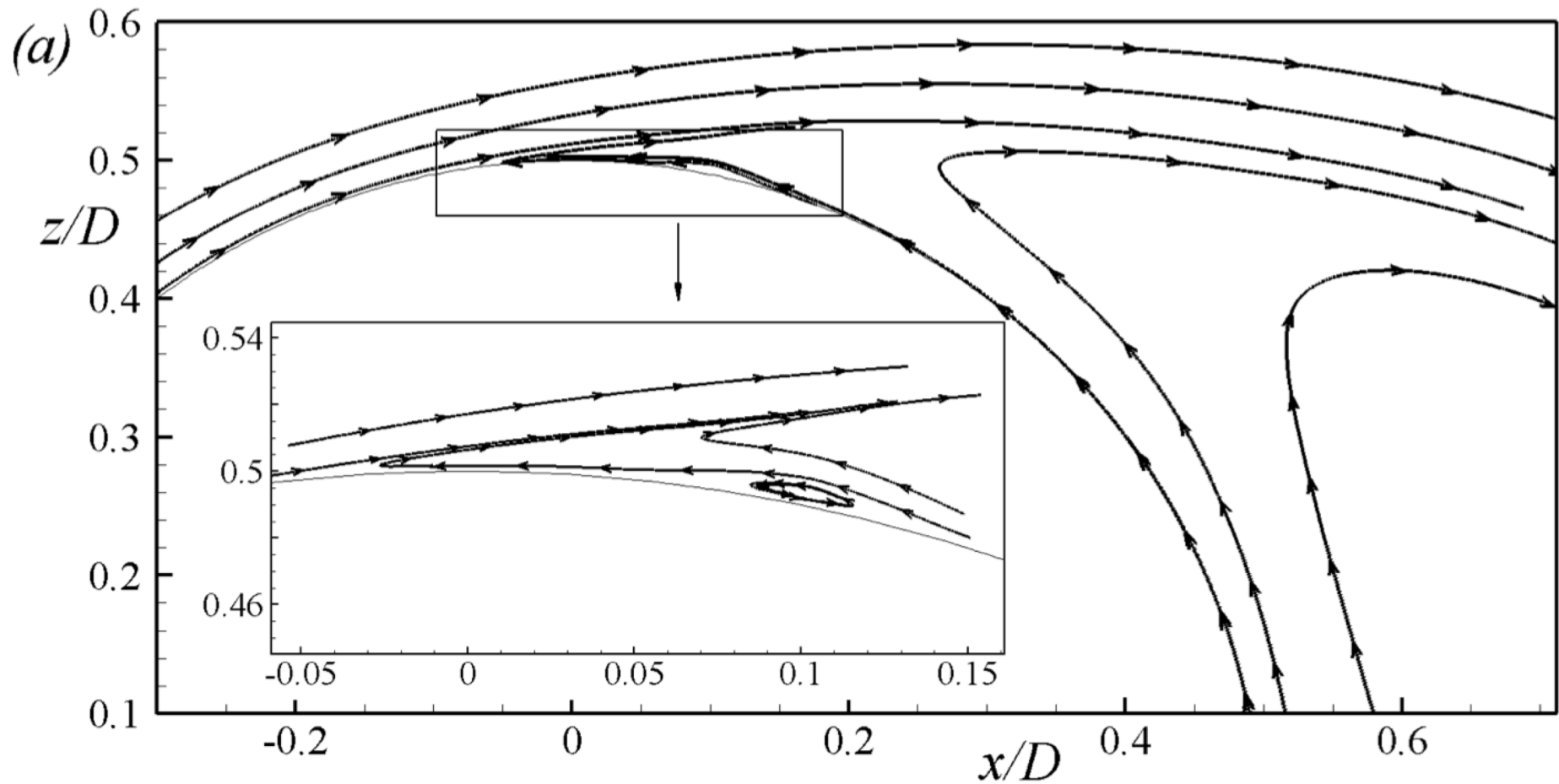


**Zero crossing indicates separation/reattachment**

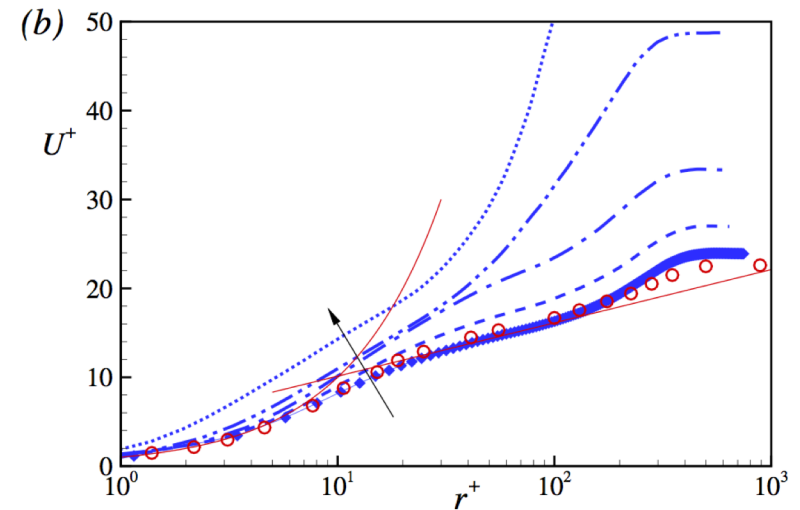
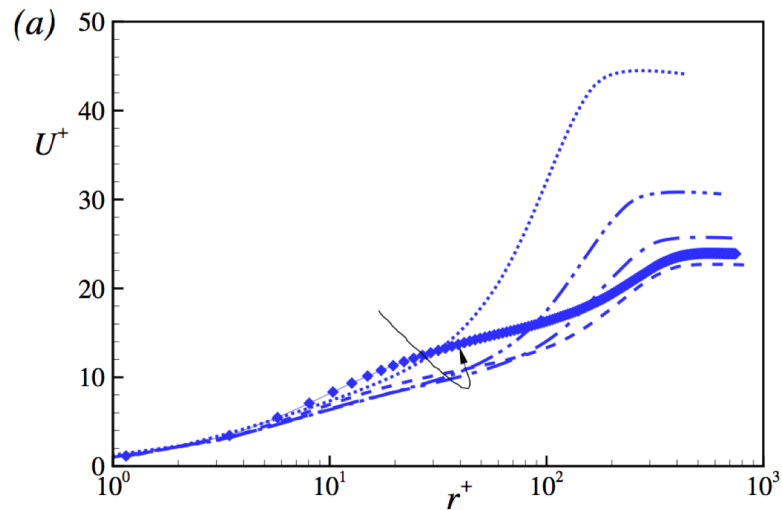
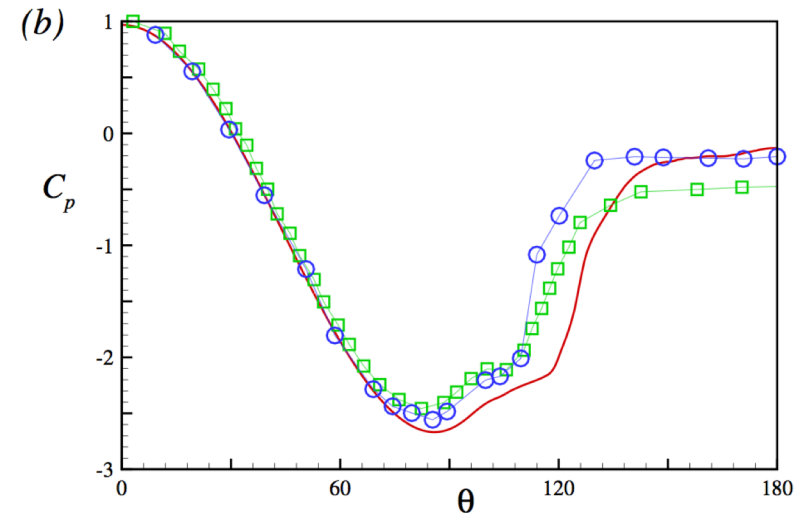
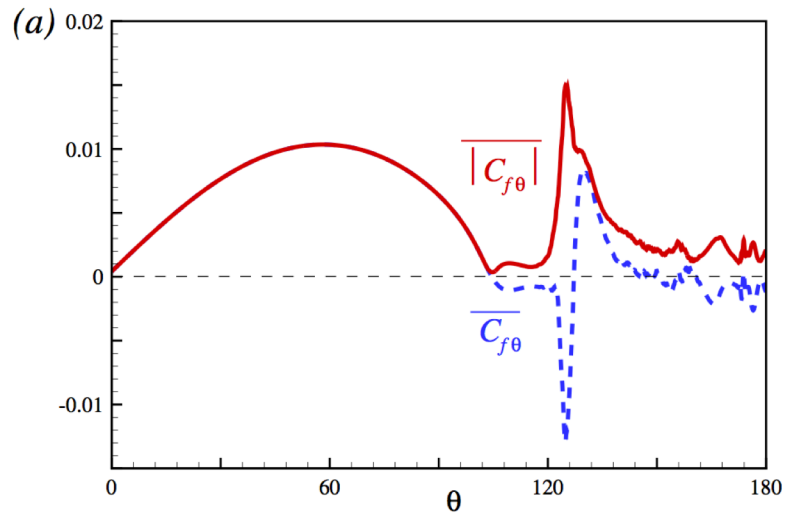
# SC: Separation: sub-critical cases



# SC: Separation: sub-critical case $Re=10^5$

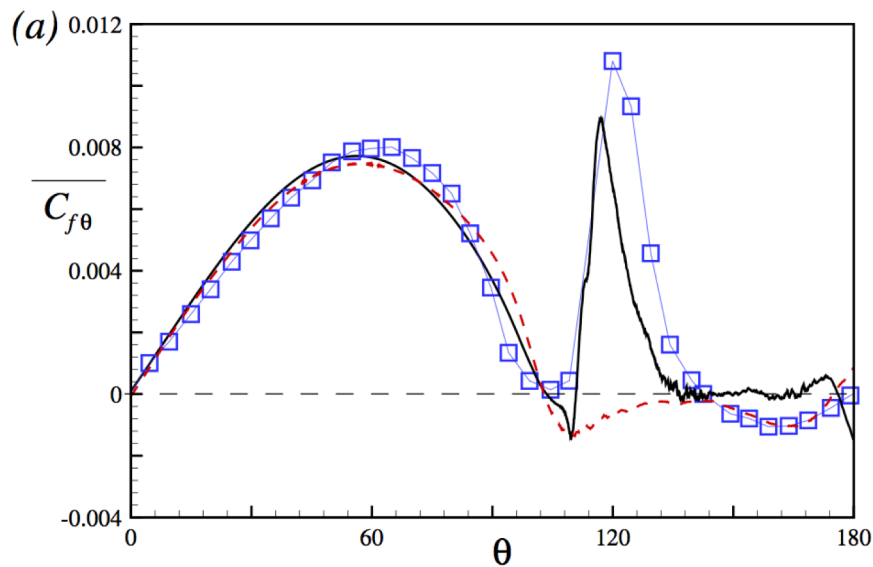


# SC: $Re = 3.5 \times 10^5$

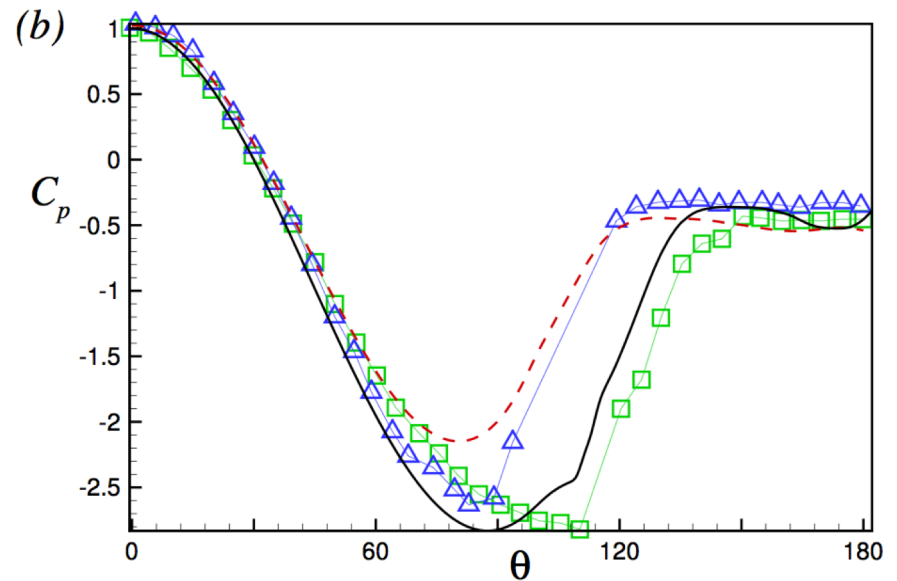


# SC: $Re = 8.5 \times 10^5$

- Coarse mesh: 2048 x 512 x 192
- Fine mesh: 8192x1024x256
- Agreement with Achenbach's experiments

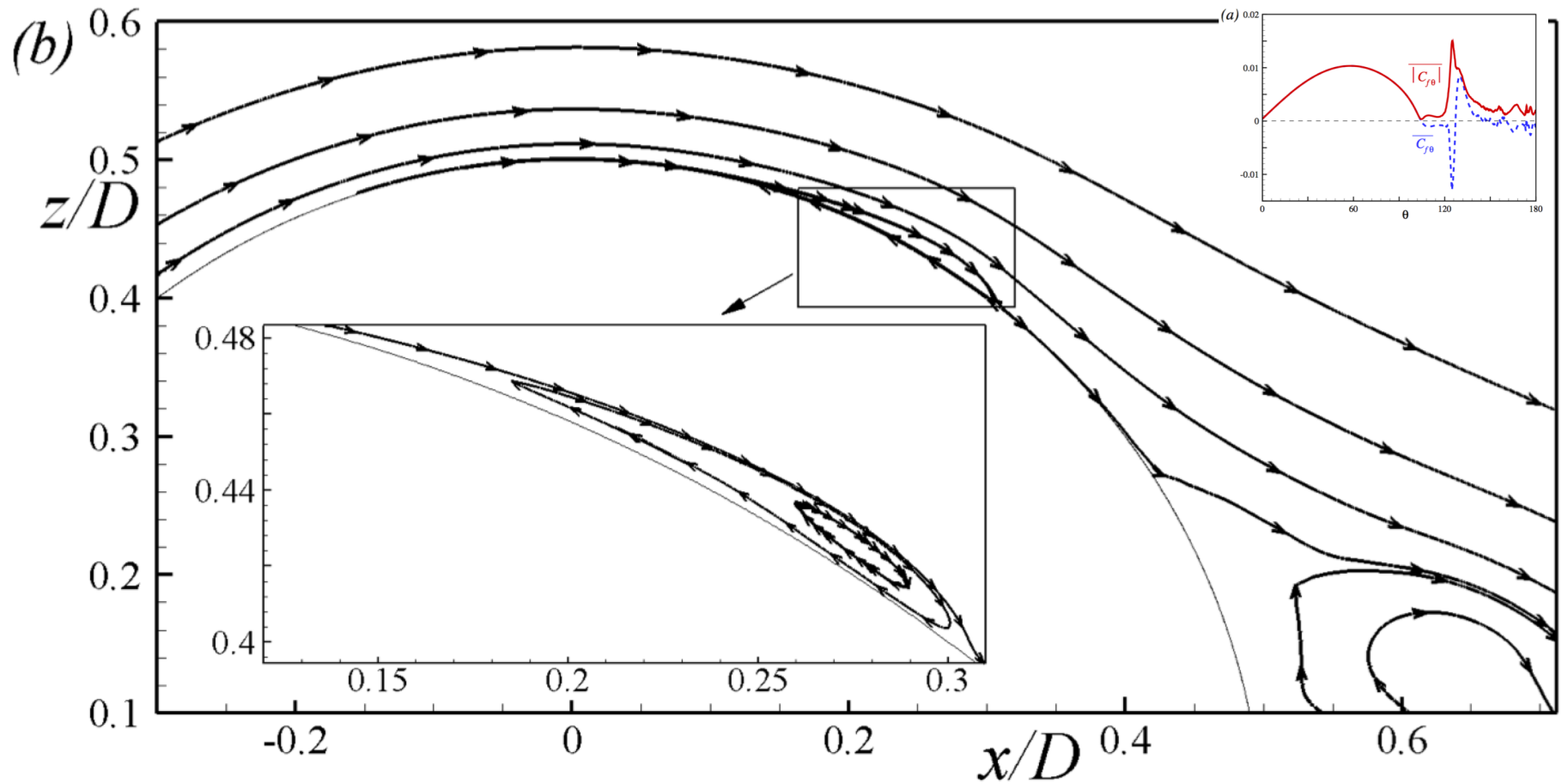


- *Skin friction coefficient*



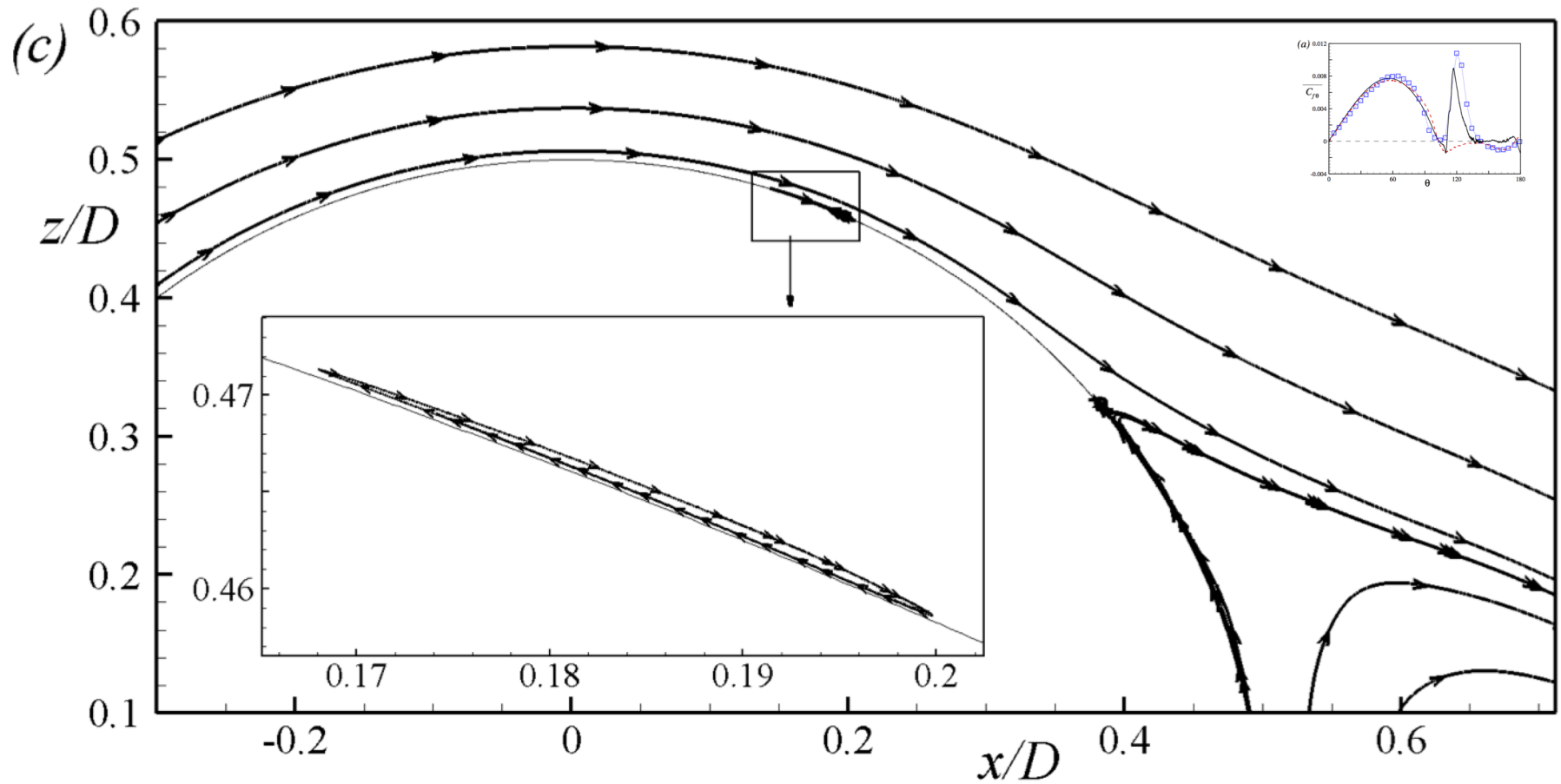
- *Pressure coefficient*

# SC: Separation, super-critical case $Re = 3.5 \times 10^5$

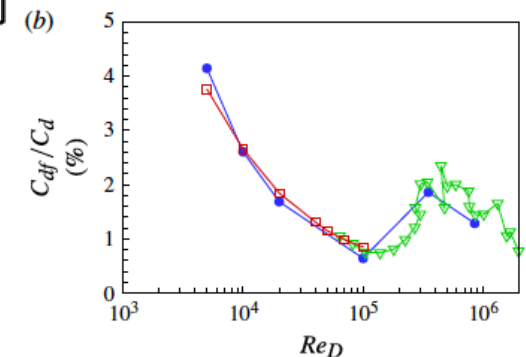
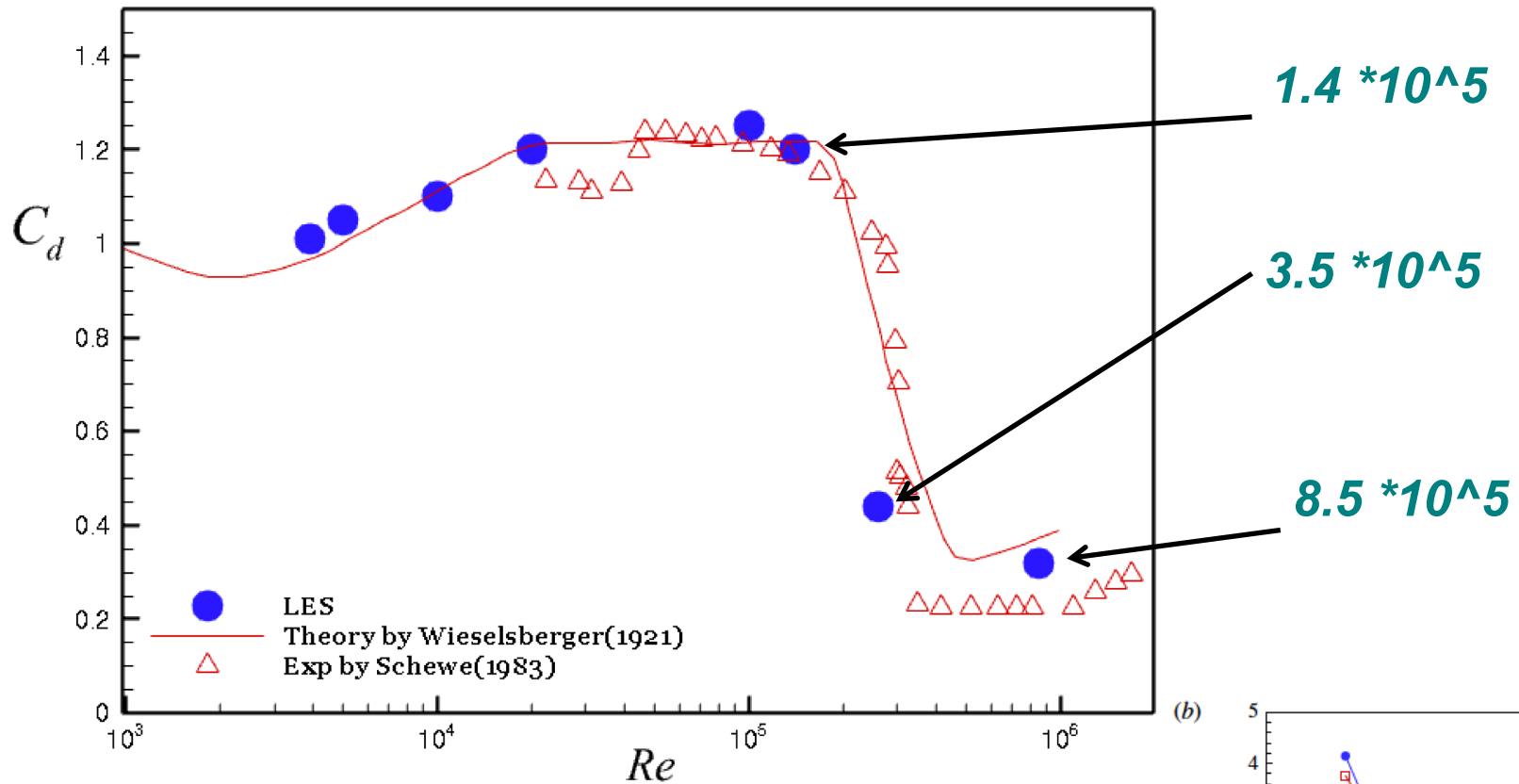




# SC: Separation, super-critical case $Re = 8.5 \times 10^5$



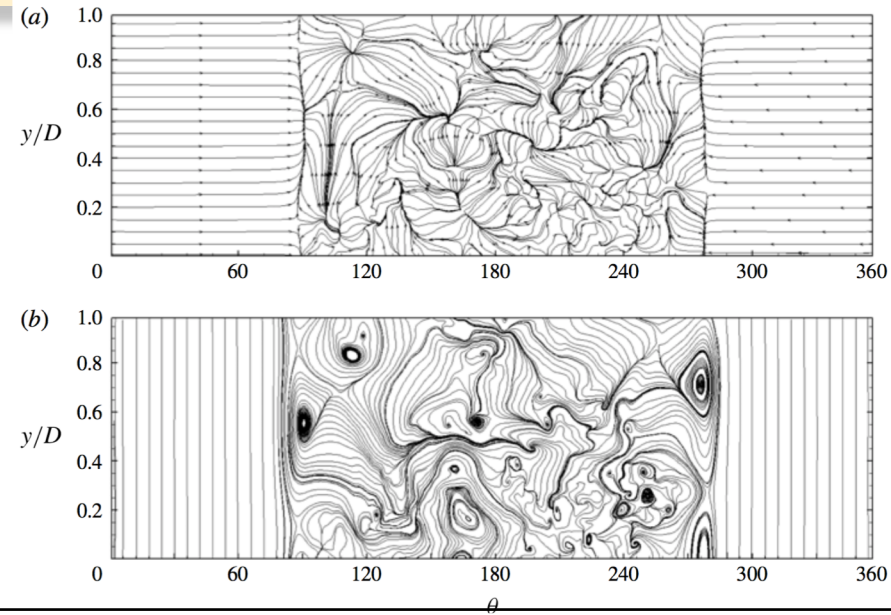
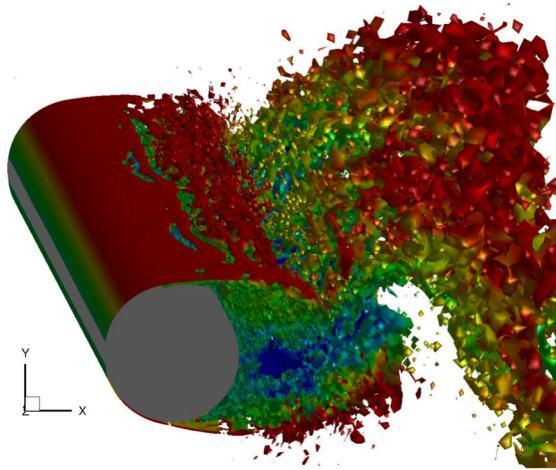
# SC: Drag Coefficient



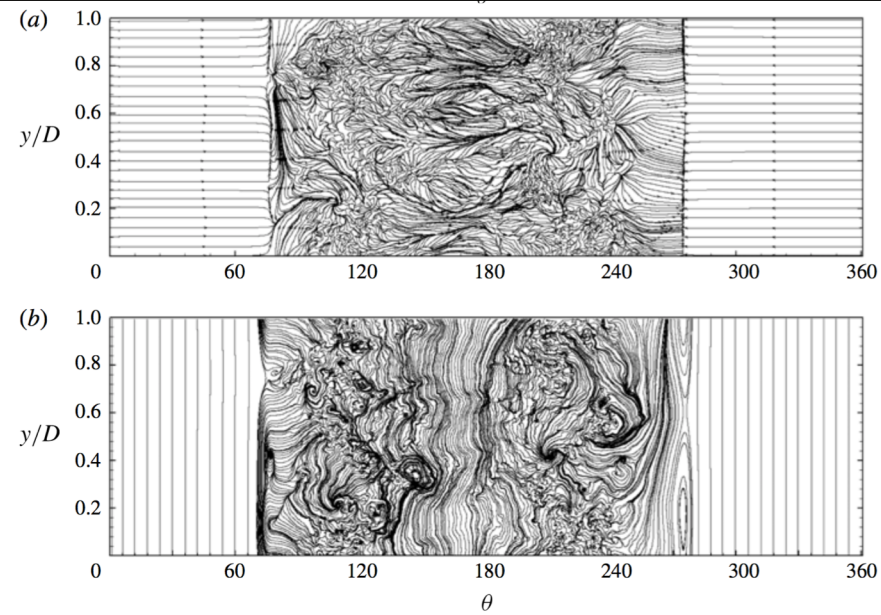
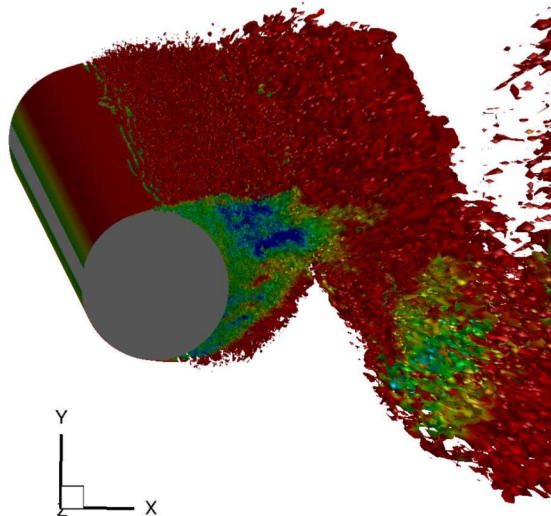
# SC: Flow past a cylinder

Cheng, Pullin, Samtaney, Zhang & Gao, JFM, 2017

$Re=10^4$



$Re=10^5$



# SC: Flow past a cylinder

- Instantaneous flow:
  - From Subcritical to supercritical  
The interaction between the shear layer induced by the primary separation and separation/reattachment bundles
- Mean flow
  - From Subcritical to supercritical  
The disappearance of secondary separation bubble and appearance of prior separation bubble
  - From supercritical to transcritical  
The disappearance of prior separation bubble
- Hypothesis: dynamic interaction between primary separation and the unsteady secondary separation could be a more general mechanism for drag crisis in bluff body flow.

# GR: Experiments: from high Re to low Re

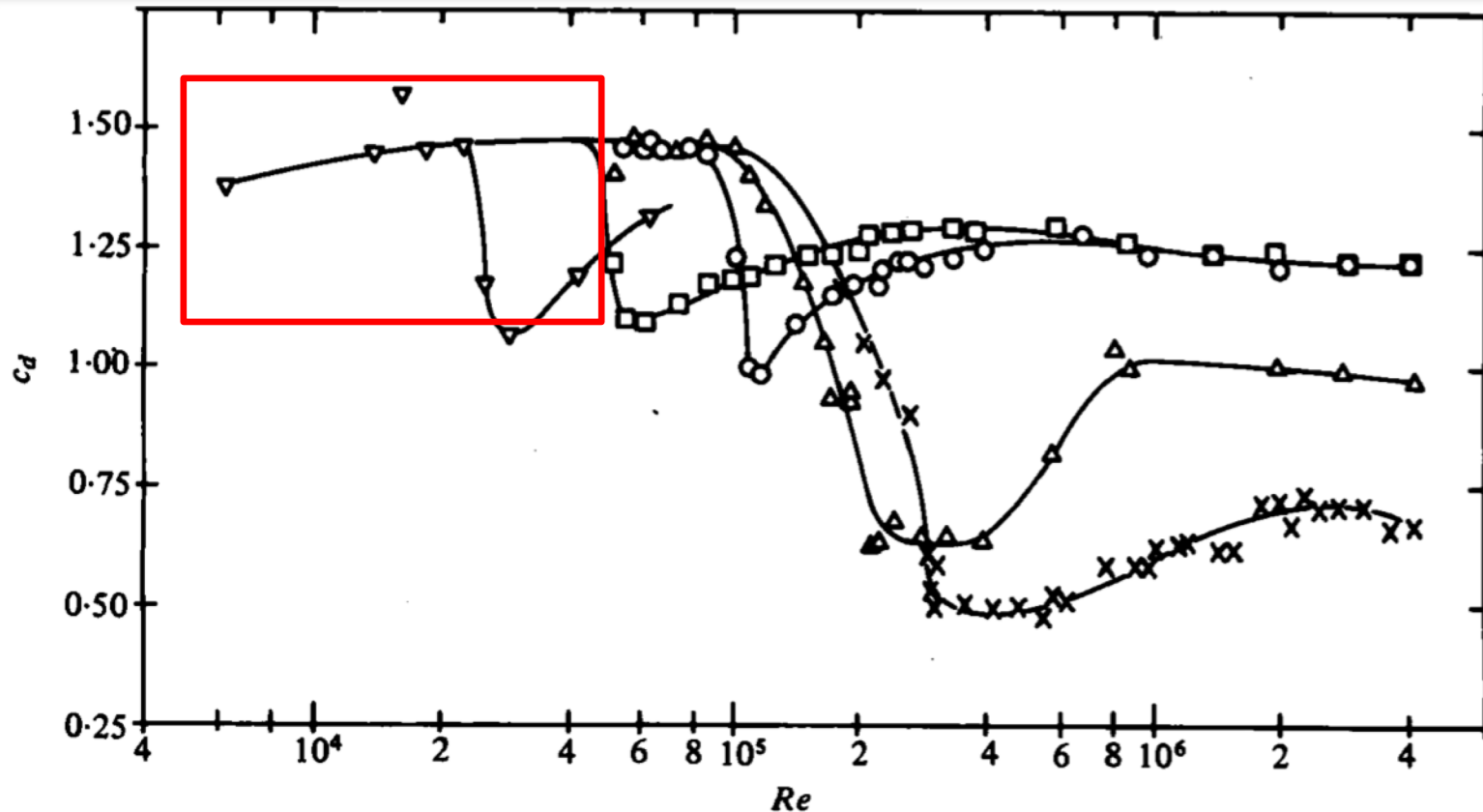


FIGURE 3. Drag coefficient of the single cylinder in cross flow at various surface roughness parameters  $k_s/d$ :  $\times$ , smooth;  $\Delta$ ,  $75 \times 10^{-5}$ ;  $\circ$ ,  $300 \times 10^{-5}$ ;  $\square$ ,  $900 \times 10^{-5}$ ;  $\nabla$ ,  $3000 \times 10^{-5}$ .

- Wall resolved LES of a cylinder with non-smooth surface

# GR: From experiment to grooved wall simulation

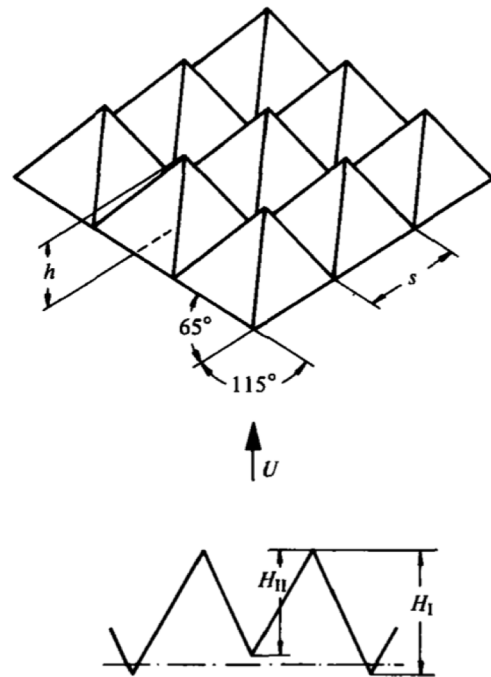
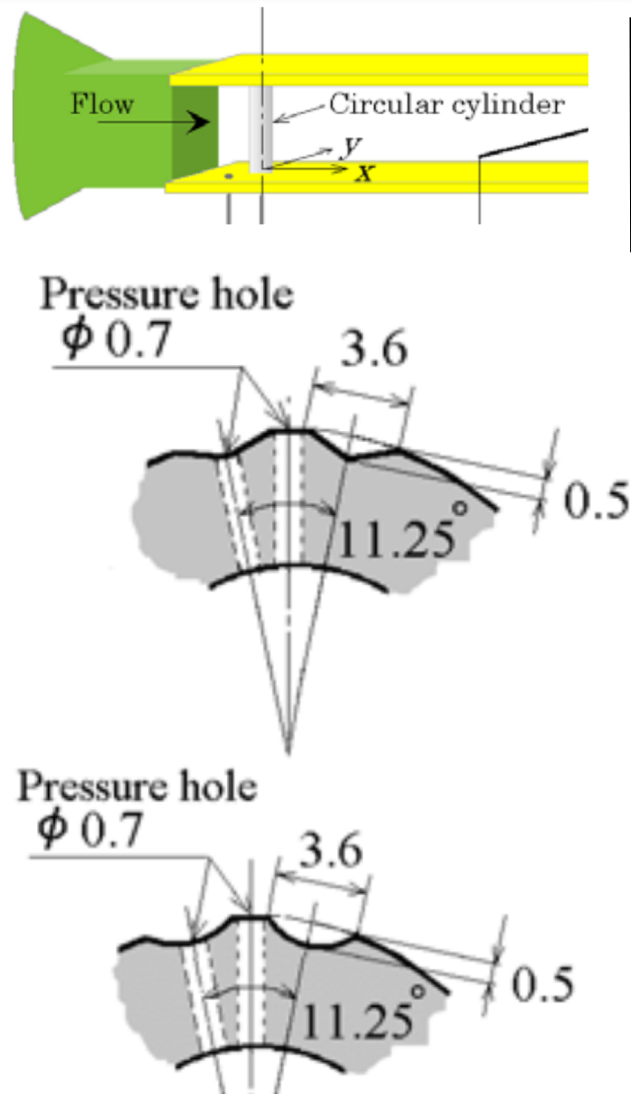
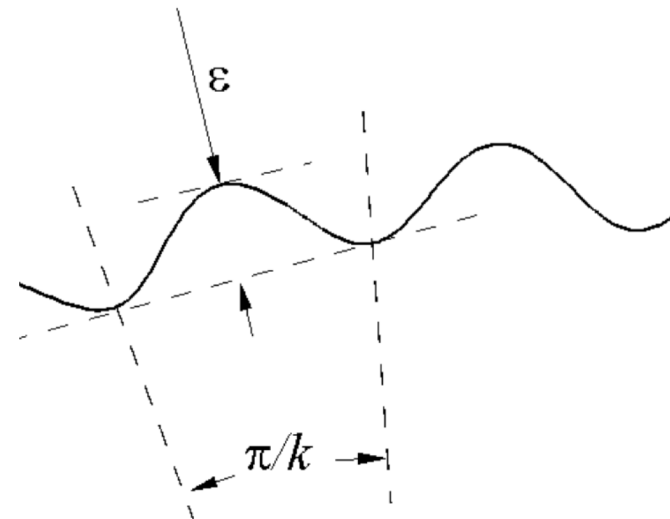


FIGURE 2. Roughness pattern.

$H_I$ (mm)	0.75–0.825
$H_U$ (mm)	0.60–0.675
$s$ (mm)	2.1
$k_s/d$	$3000 \times 10^{-5}$



## • Simulation setup

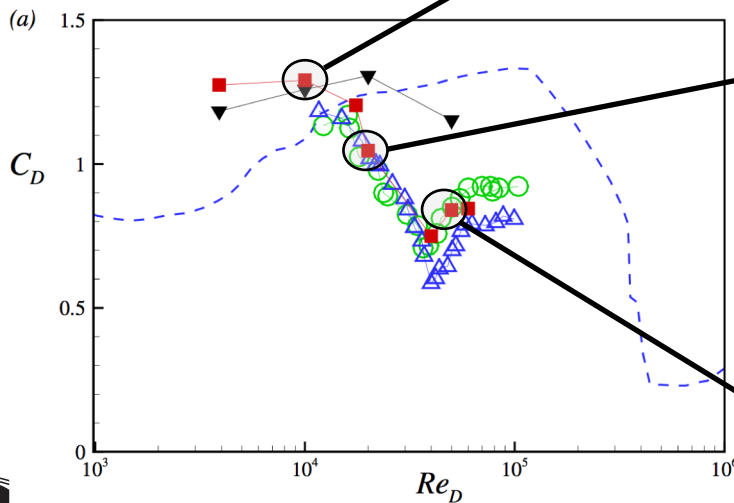
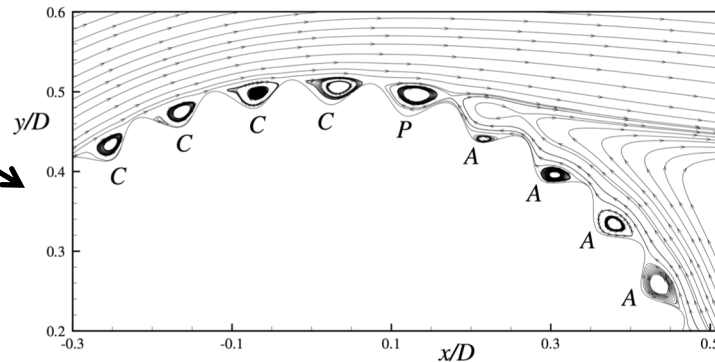
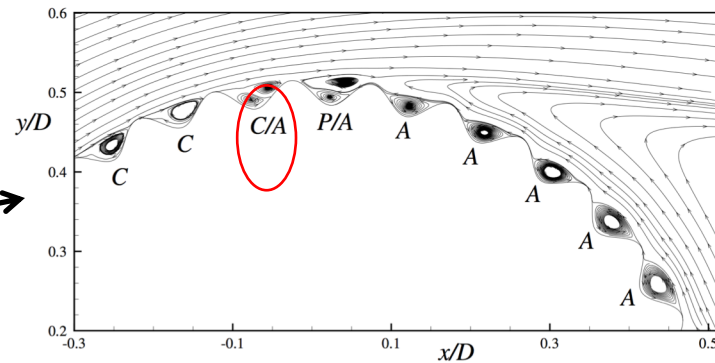
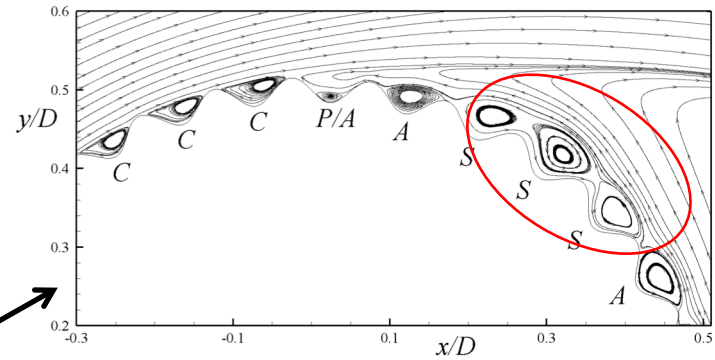
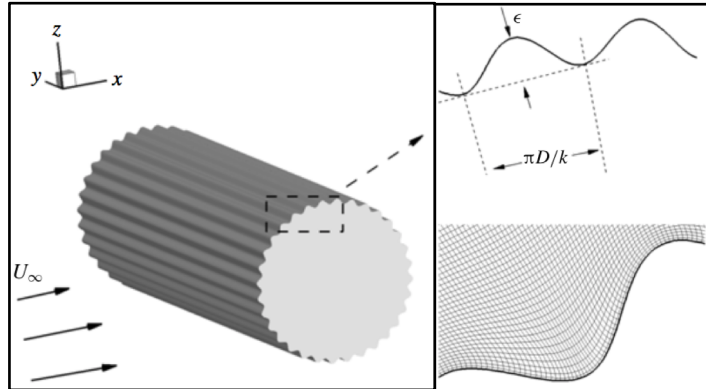


$$k = 32$$

$$\epsilon = 1/k$$

# GR: Flow past a grooved cylinder

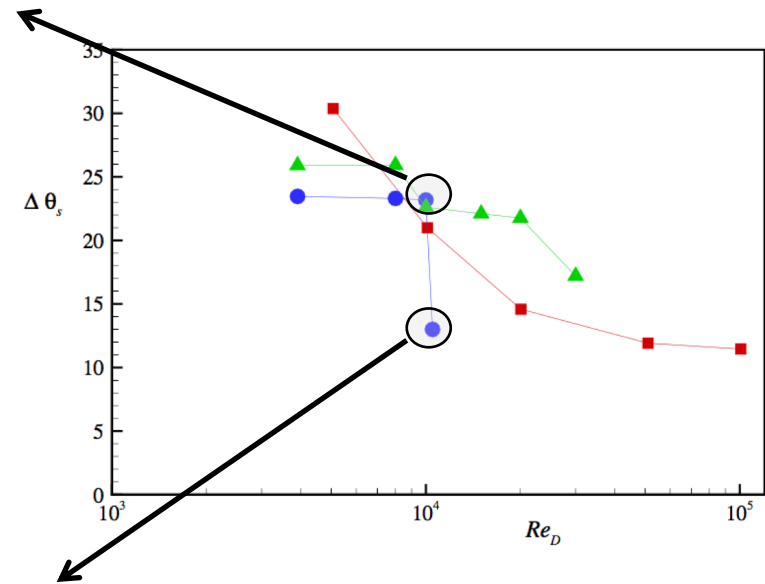
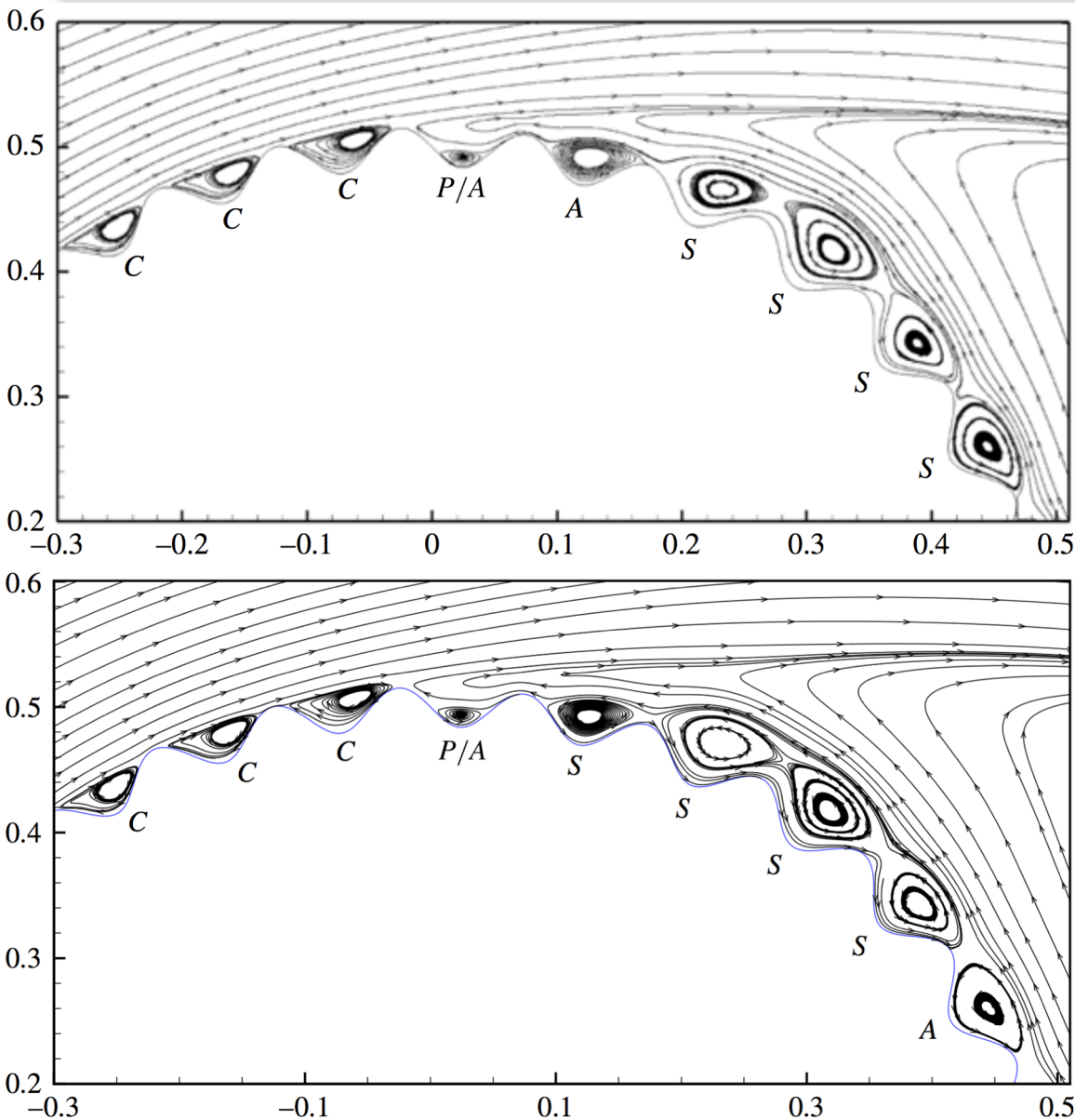
Cheng, Pullin, Samtaney, JFM, 2018





# GR: Secondary separation bubble

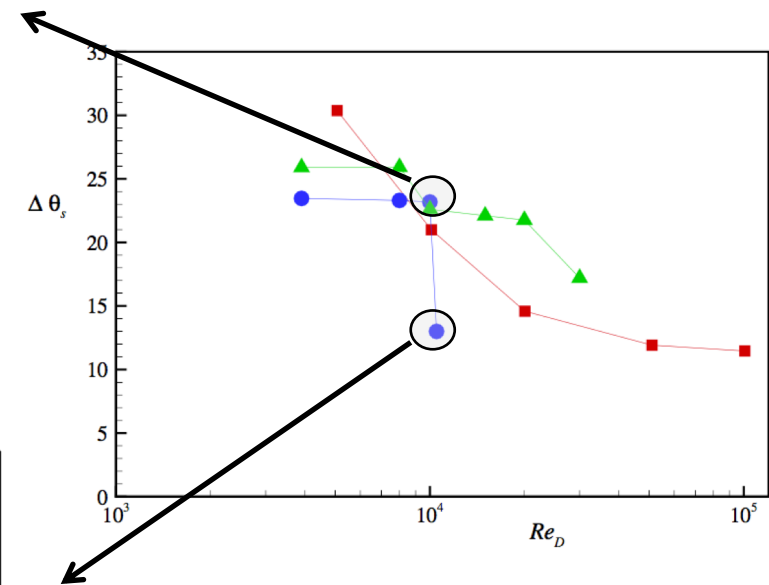
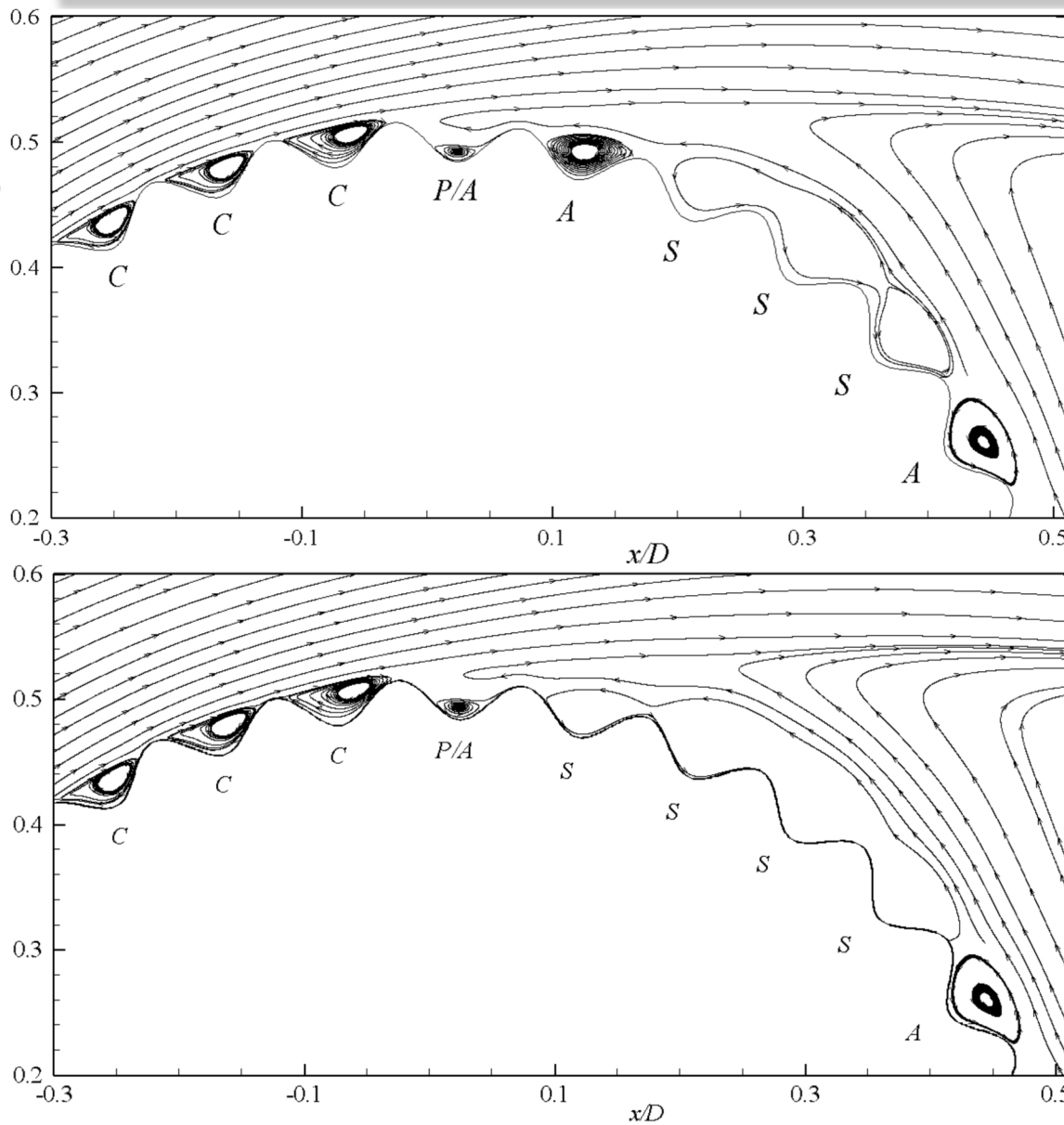
Cheng, Pullin, Samtaney, JFM, 2018





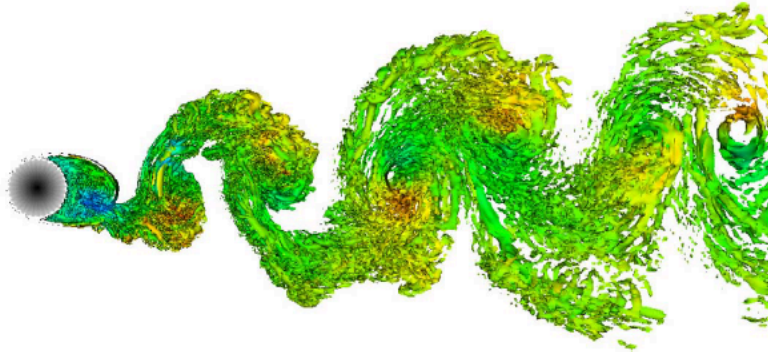
# GR: Secondary separation bubble

Cheng, Pullin, Samtaney, JFM, 2018

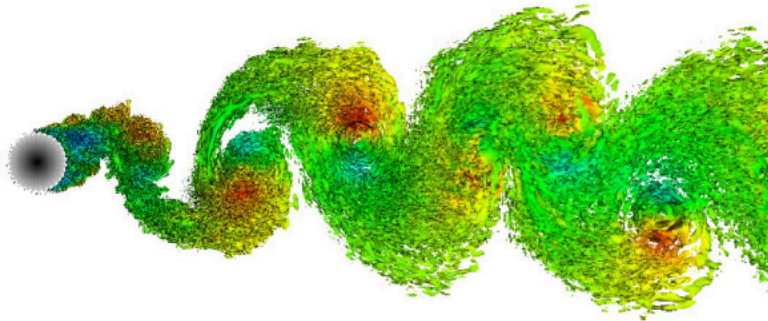


## GR: Instantaneous isosurfaces of $Q$

(a)  $Re=3900$

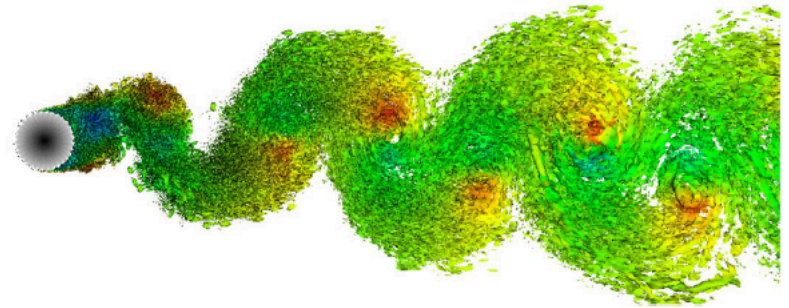


(b)

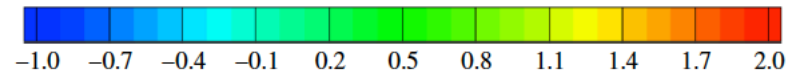
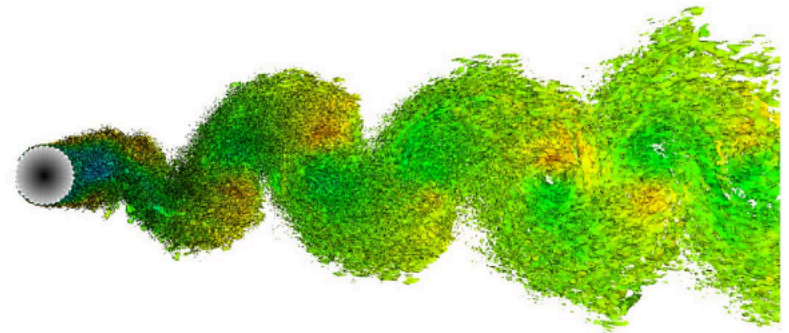


$Re=10^4$

(c)  $Re=2 \times 10^4$



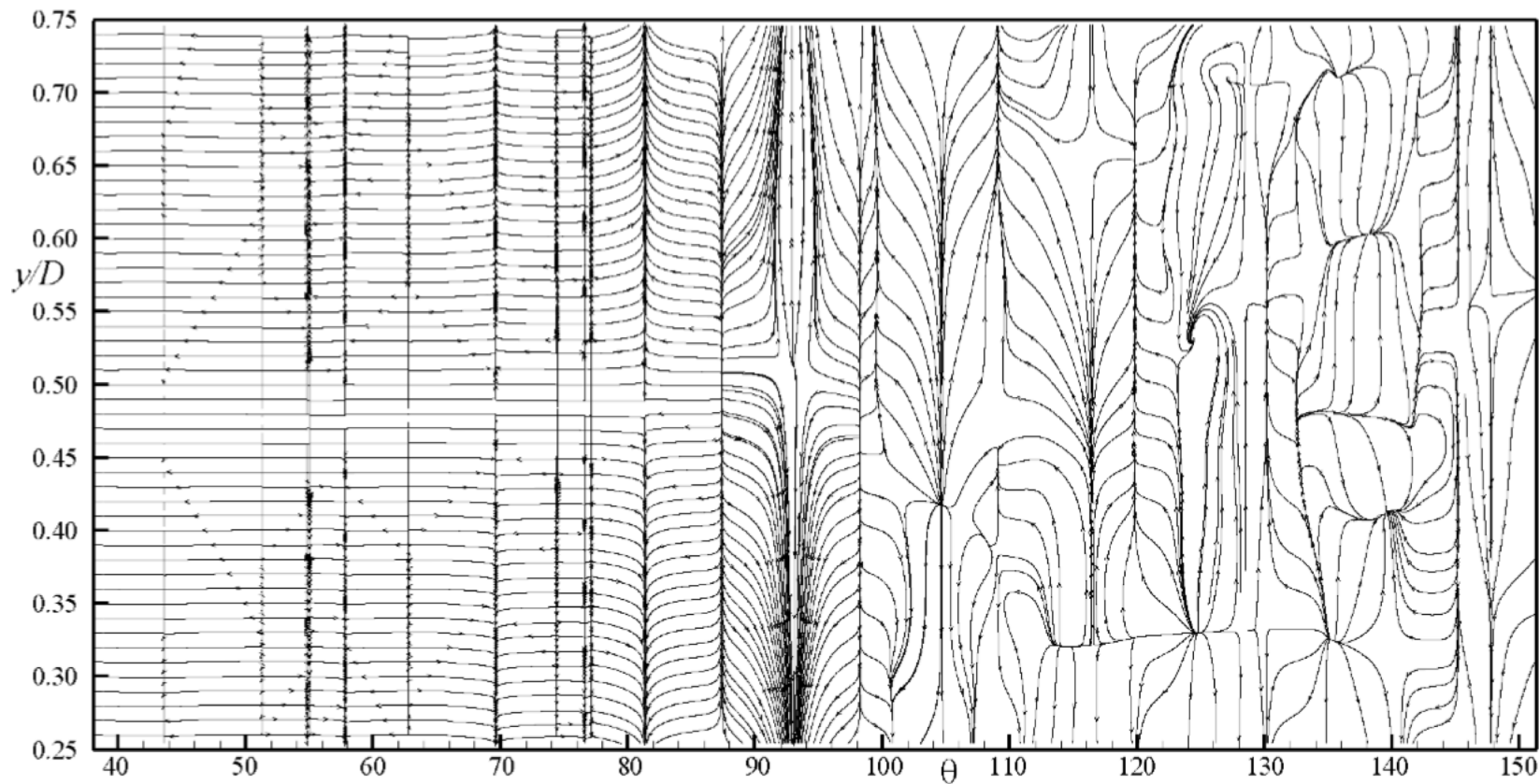
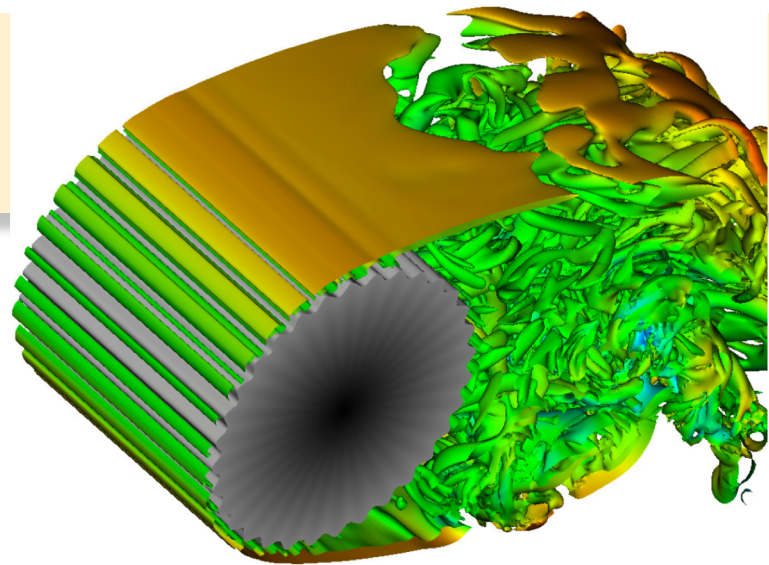
(d)



$Re=5 \times 10^4$

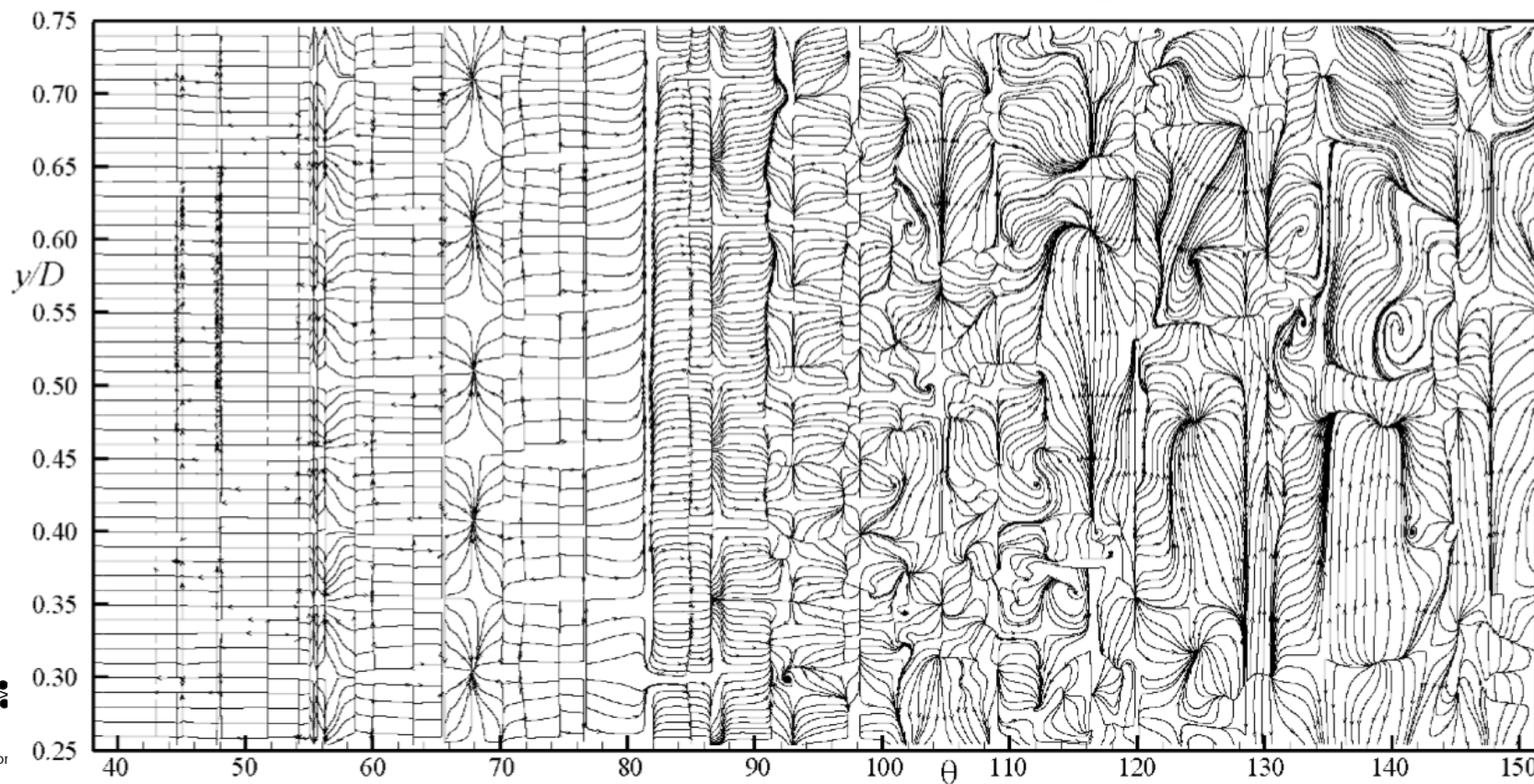
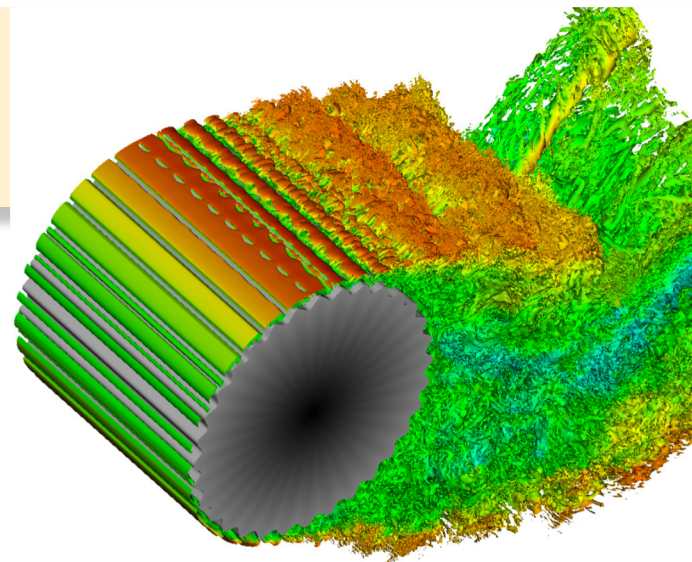
**GR:**

$$Re_D = 3.9 \times 10^3$$



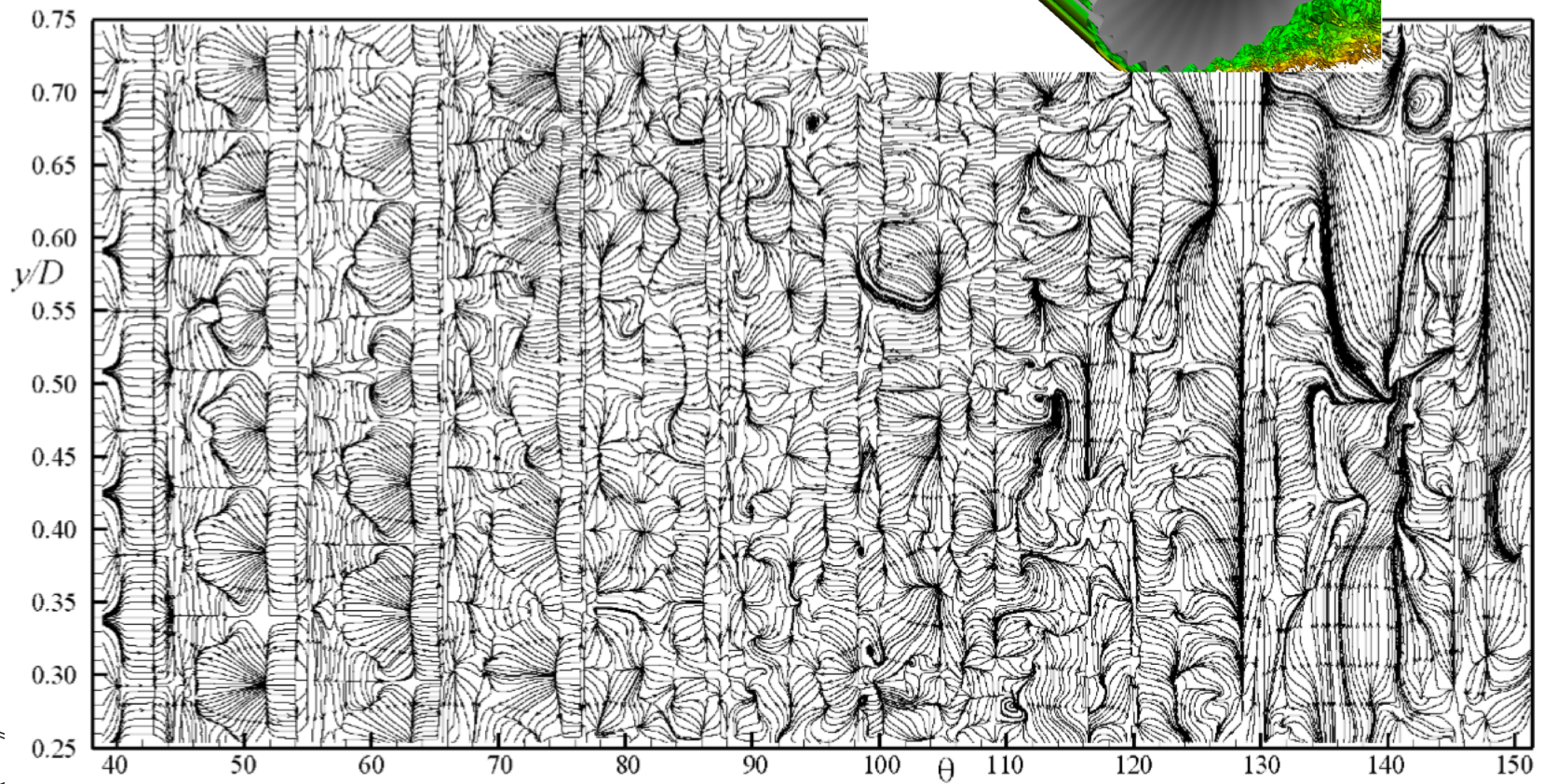
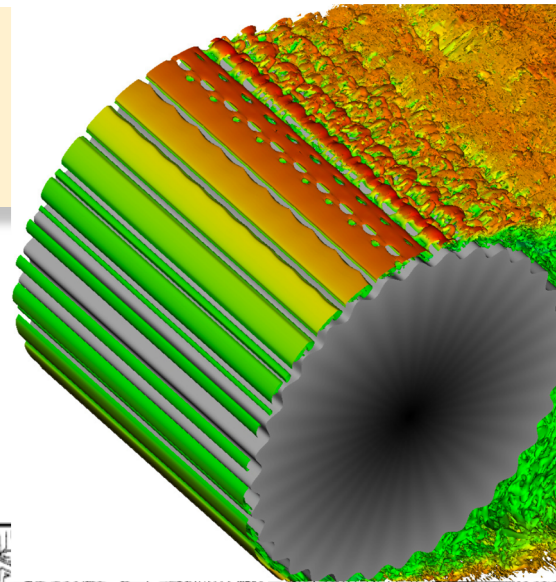


**GR:**  
 $Re_D = 1 \times 10^4$





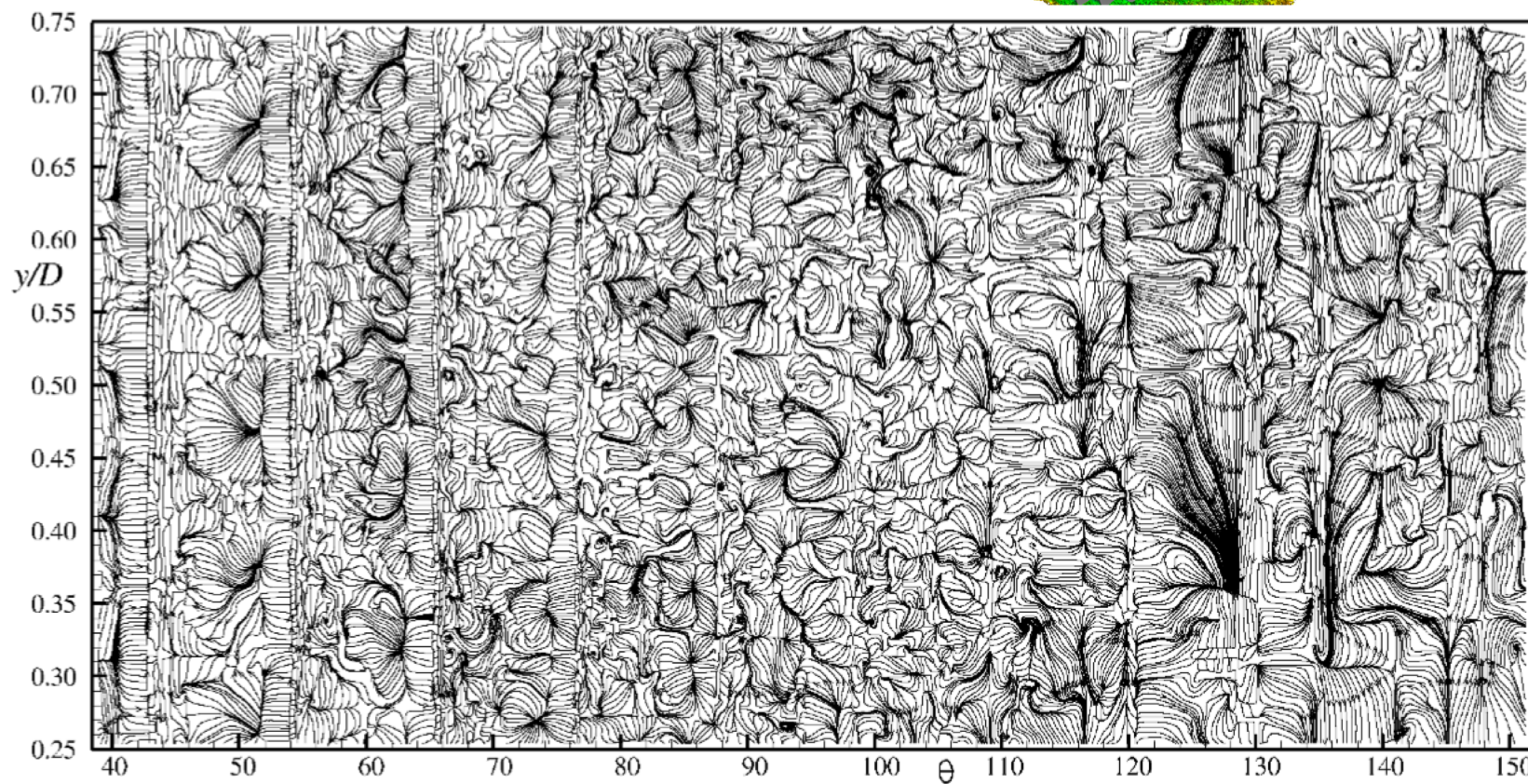
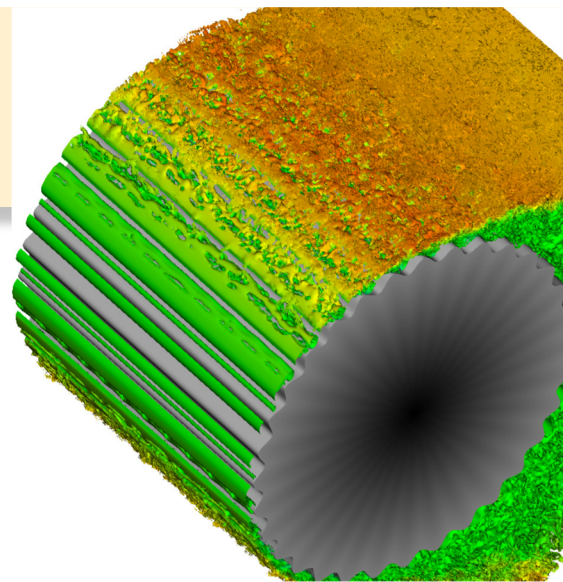
*GR:*  
 $Re_D = 2 \times 10^4$





**GR:**

$$Re_D = 5 \times 10^4$$

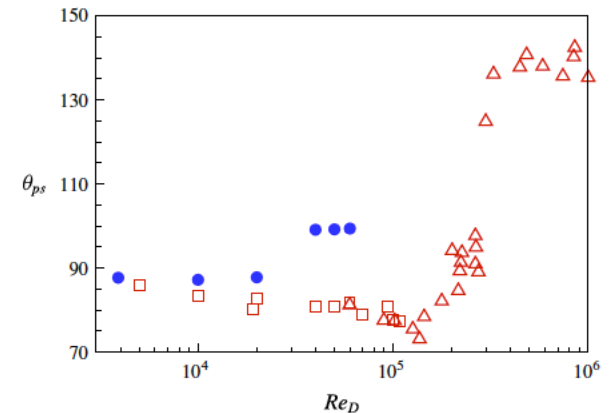


# Cylinder + Grooved cylinder

- Cylinder flow

Hypothesis: drag crisis is due to dynamic interaction between primary separation with unsteady secondary separation.

- **Mean flow:** Secondary separation bubble
- **Instantaneous:** separatrices and diverging bundles with small-scale separation/reattachment cells



- Grooved Cylinder flow

Confirm: drag crisis is also due to dynamic interaction between primary separation with unsteady secondary separation.

- **Mean flow:** Secondary separation bubble
- **Instantaneous:** separatrices and diverging bundles with small-scale separation/reattachment cells

# Smooth Cylinder + Grooved cylinder

- Smooth Cylinder flow

Hypothesis: drag crisis is due to dynamic interaction between primary separation with unsteady secondary separation.

- **Mean flow:** Secondary separation bubble
- **Instantaneous:** separatrices and diverging bundles with small-scale separation/reattachment cells
- The secondary separation bubble moves upstream **continuously**

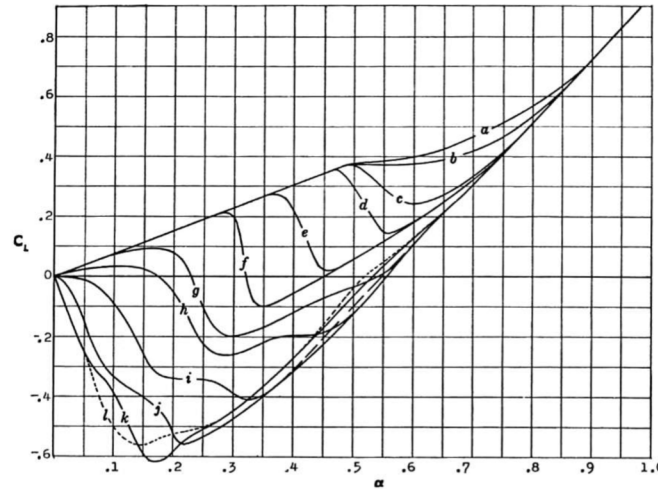
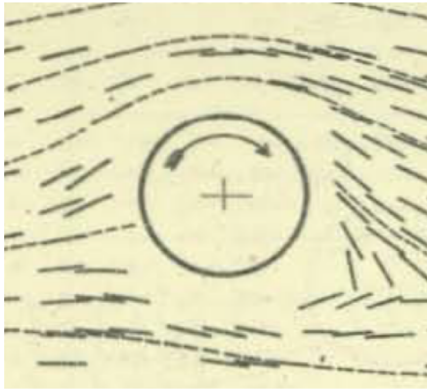
- Grooved Cylinder flow

Confirm: drag crisis is also due to dynamic interaction between primary separation with unsteady secondary separation.

- **Mean flow:** Secondary separation bubble
- **Instantaneous:** separatrices and diverging bundles with small-scale separation/reattachment cells
- The secondary separation bubble moves upstream **discontinuously**



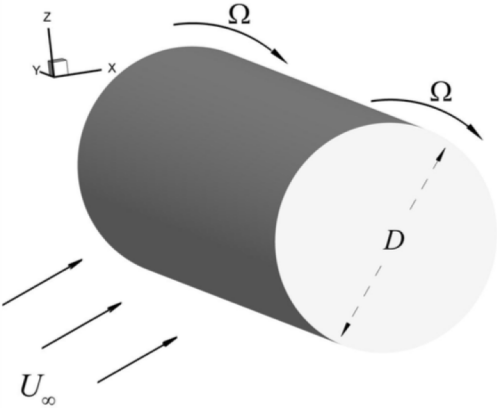
# Background of Flow past a rotating cylinder



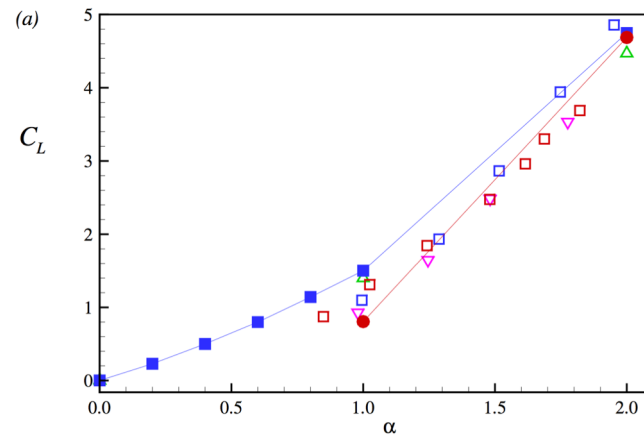
Curve designation	Reynolds no.
a	3.58 $\times 10^4$
b	4.9
c	6.07
d	7.91
e	9.9
f	12.8
g	15.2
h	18.15
i	22.5
j	26
k	29.5

# RC: Flow past a rotating cylinder

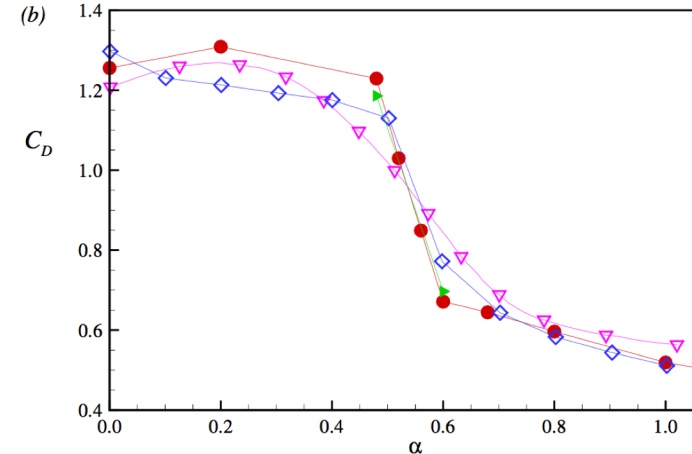
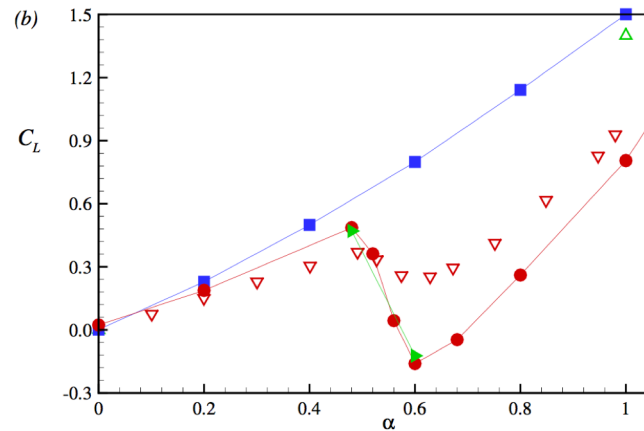
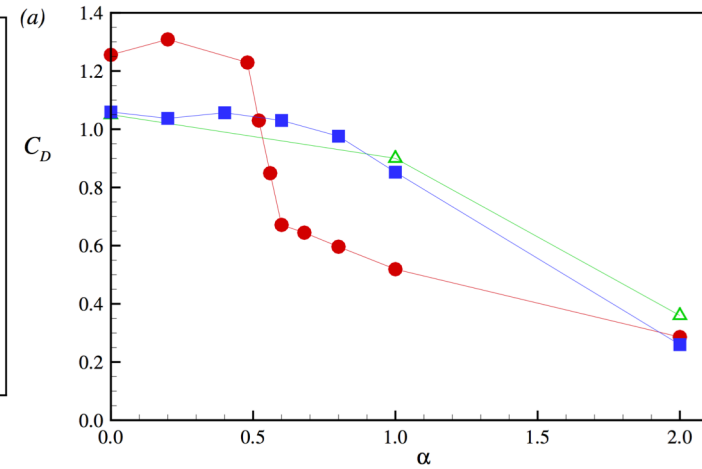
Cheng, Pullin, Samtaney, JFM 2018



*Lift Coefficient*



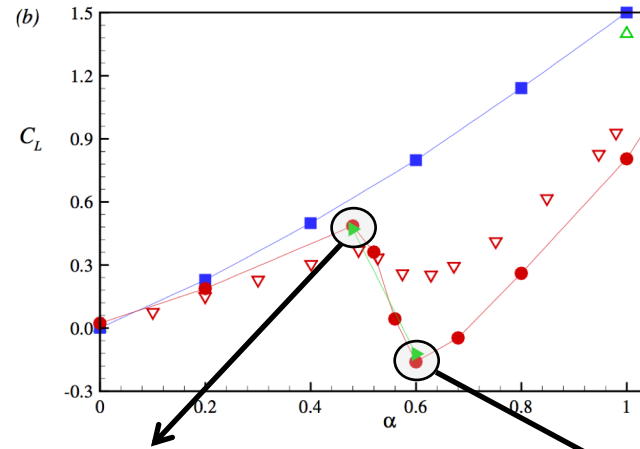
*Drag Coefficient*



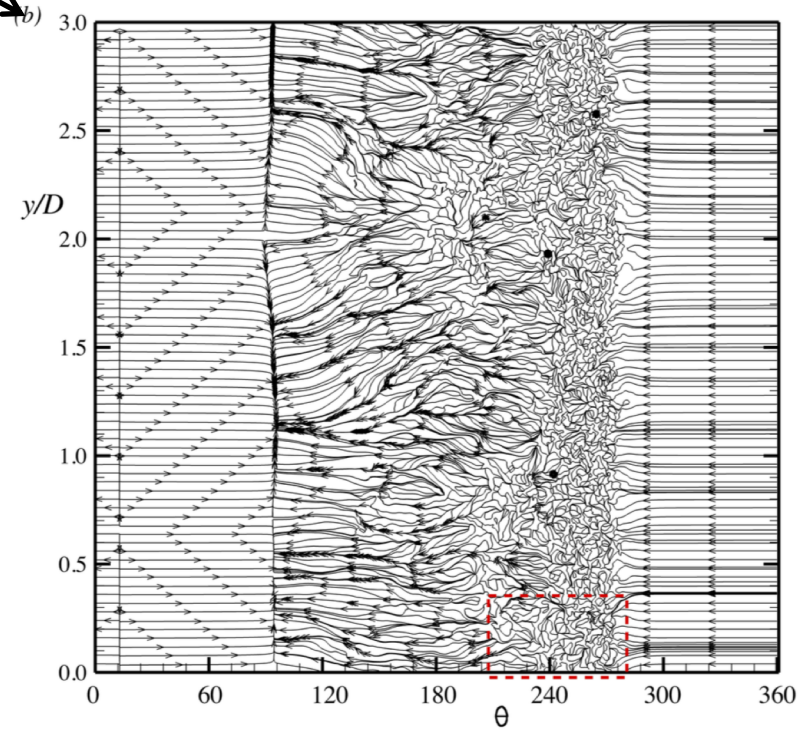
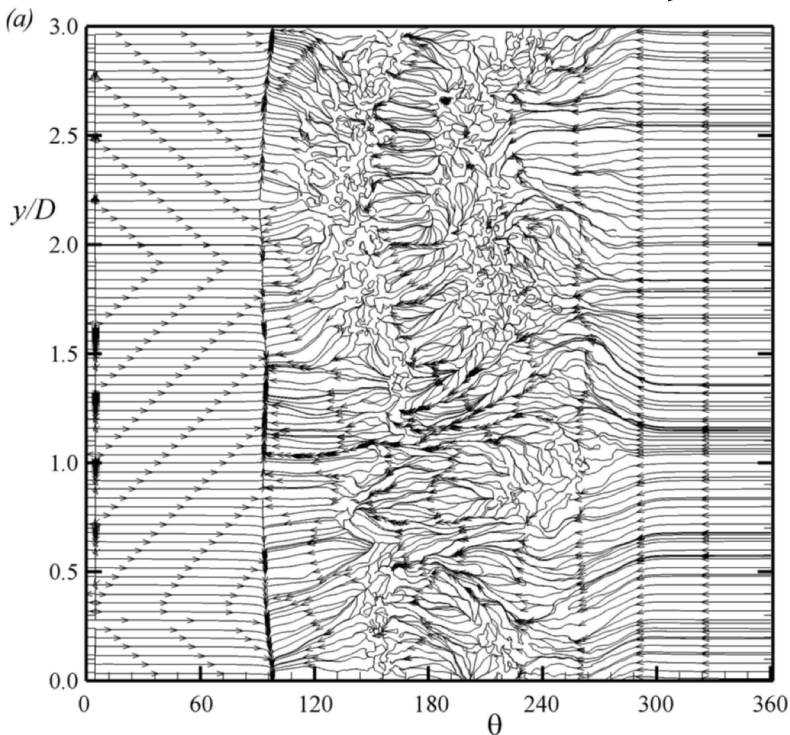
# RC: Flow past a rotating cylinder

Cheng, Pullin, Samtaney, JFM 2018

*Distributed small  
scale  
separation/reattach  
ment cells*



*aggregated small  
scale  
separation/reattach  
ment cells*

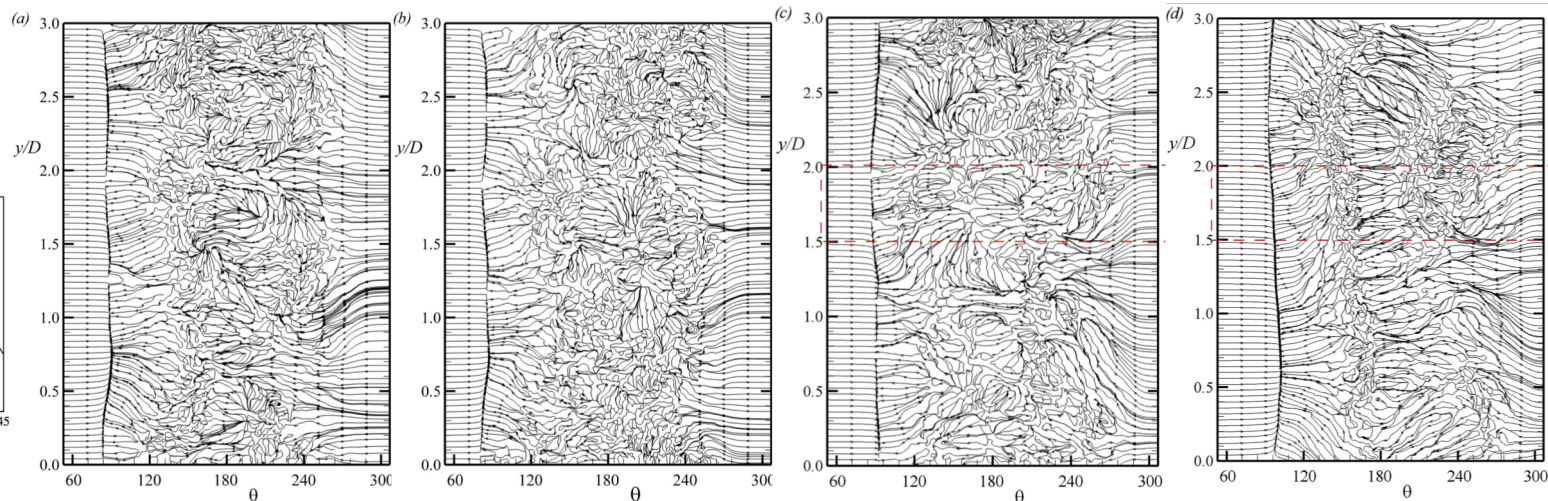
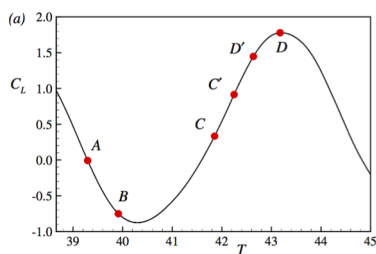




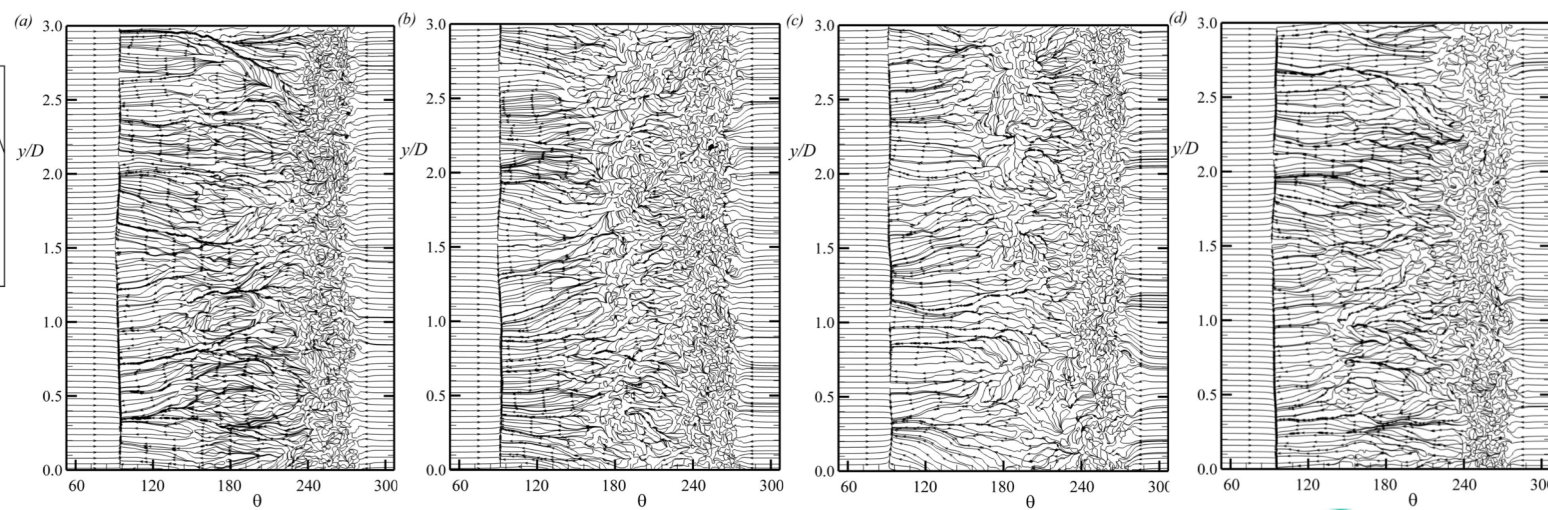
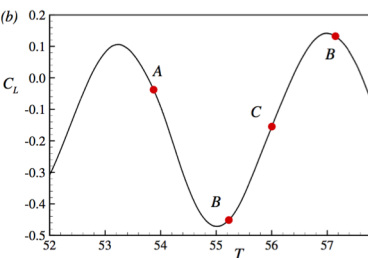
# RC: Flow past a rotating cylinder

Cheng, Pullin, Samtaney, JFM 2018

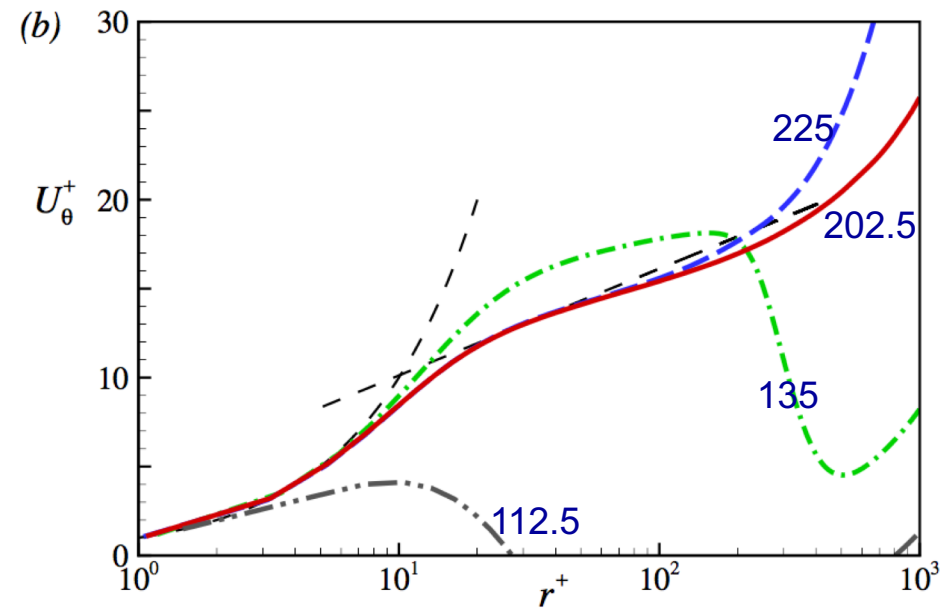
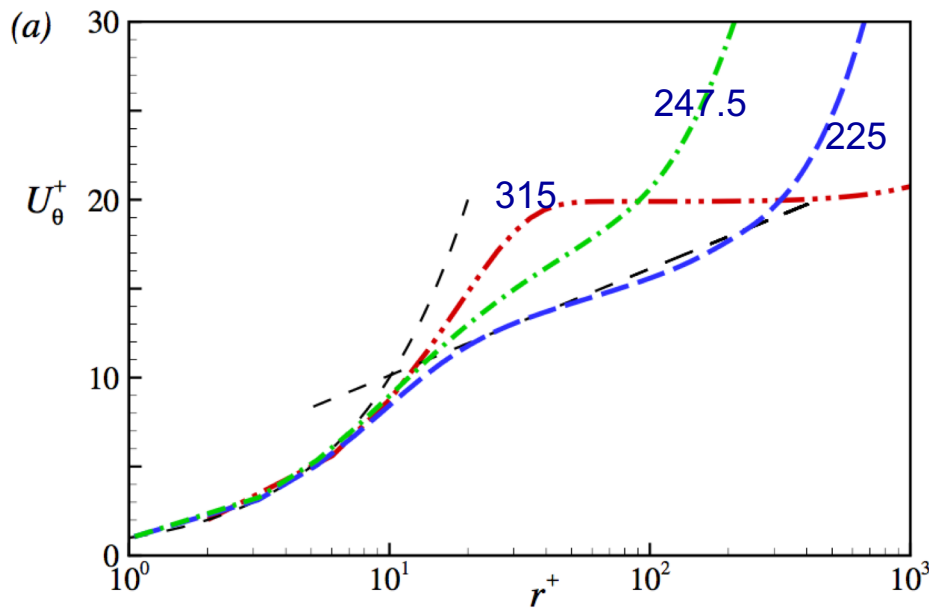
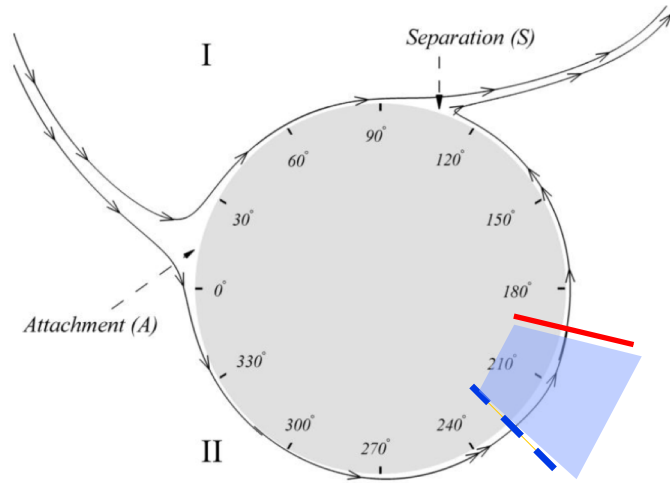
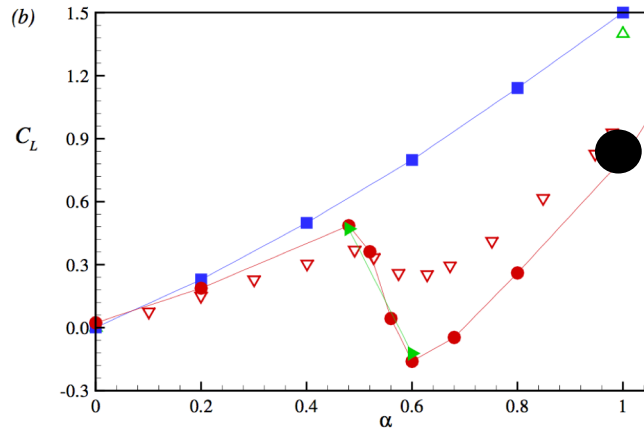
$$\alpha = 0.48$$



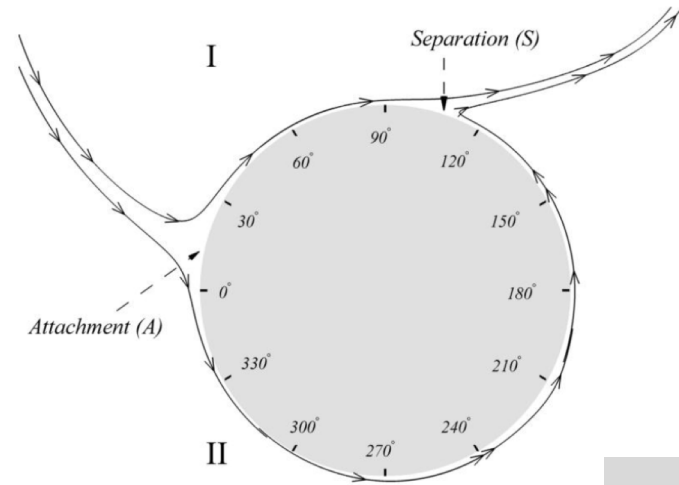
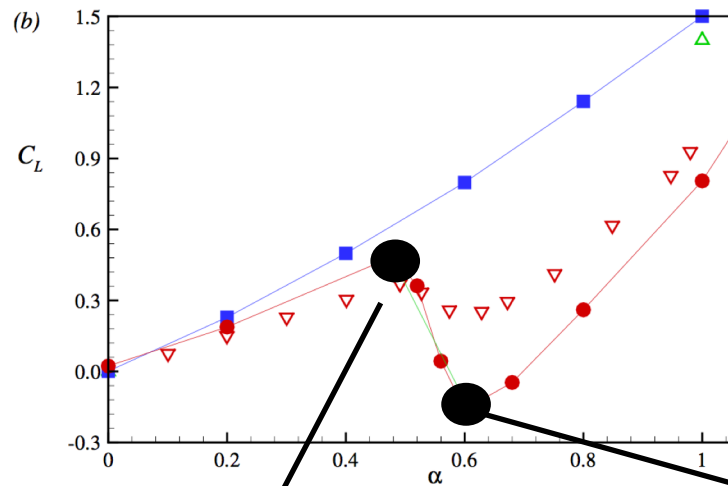
$$\alpha = 0.6$$



# RC: turbulence observed at $\alpha=1.0$

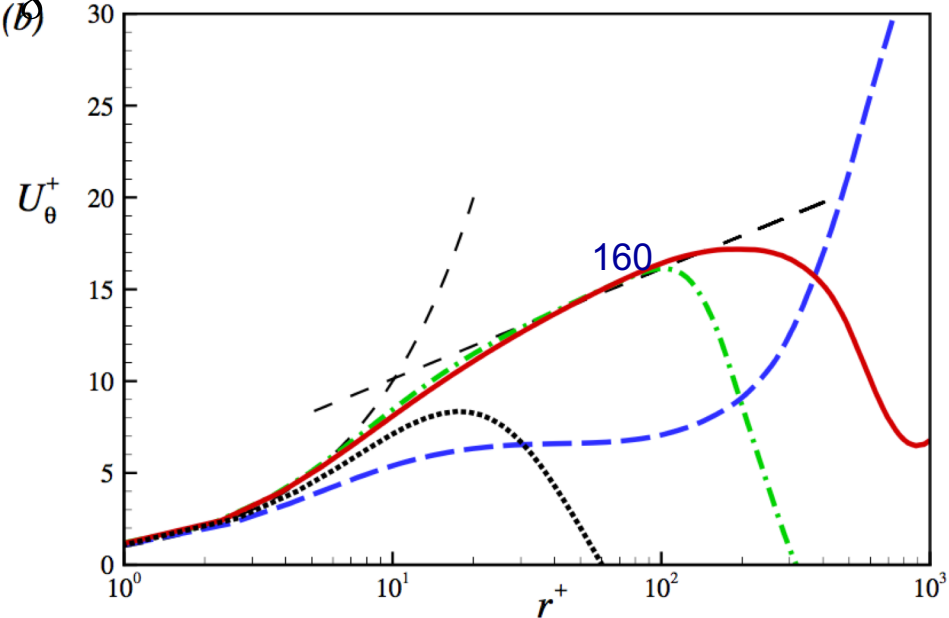


# RC: No difference in turbulence across crisis



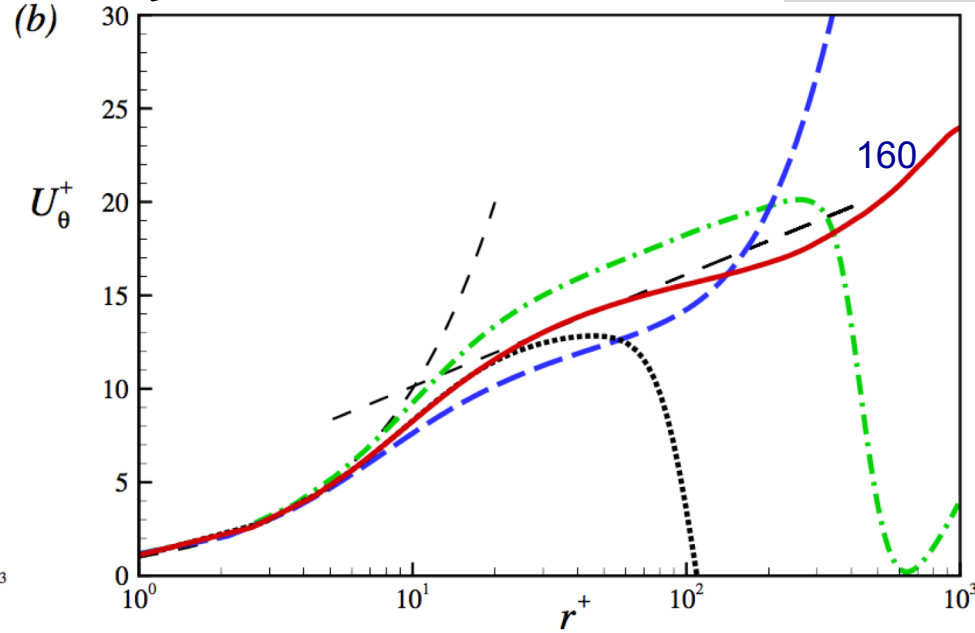
$\alpha=0.4$

(b)



$\alpha=0.6$

(b)



# Cylinder + Grooved cylinder + Rotating cylinder

- Cylinder flow

Hypothesis: dynamic interaction between unsteady separations could be a more general mechanism for drag crisis in bluff body flow.

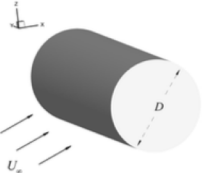
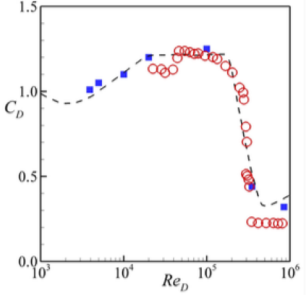
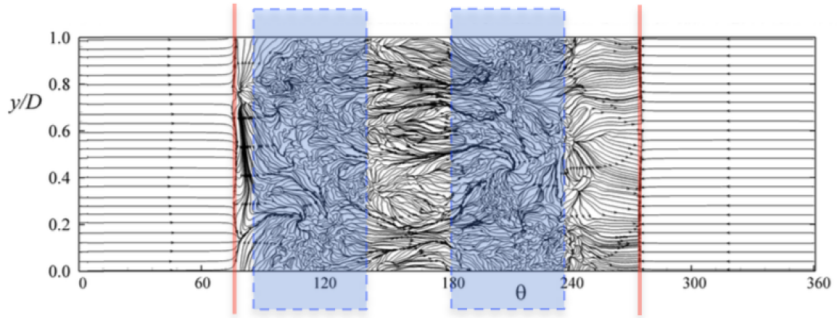
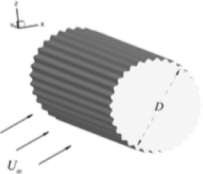
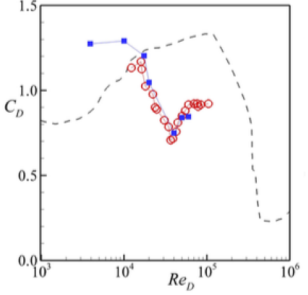
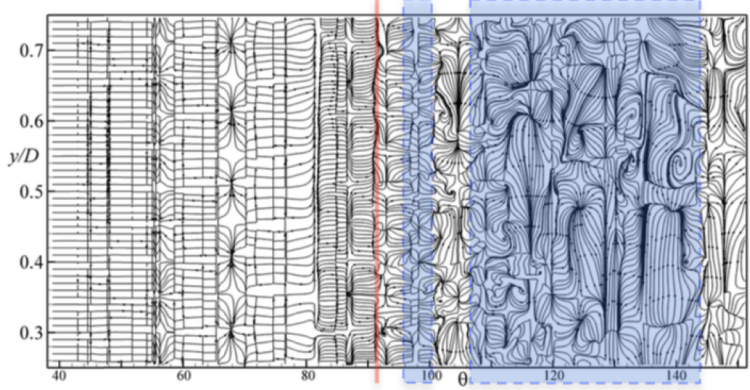
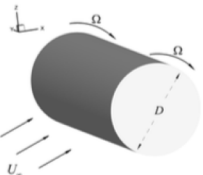
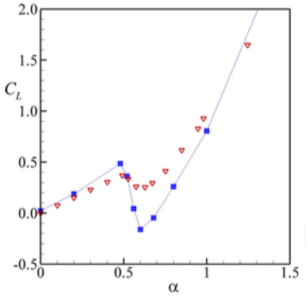
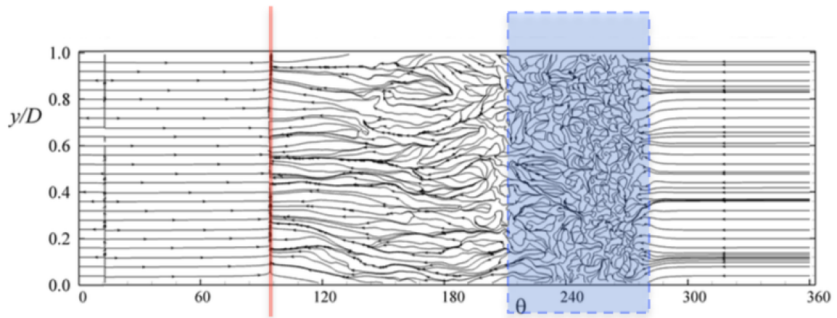
- Grooved cylinder

- Dynamic interaction between unsteady separations is still important.
- Mean flow: still observe the secondary separation bubble and prior separation bubble

- Rotating cylinder

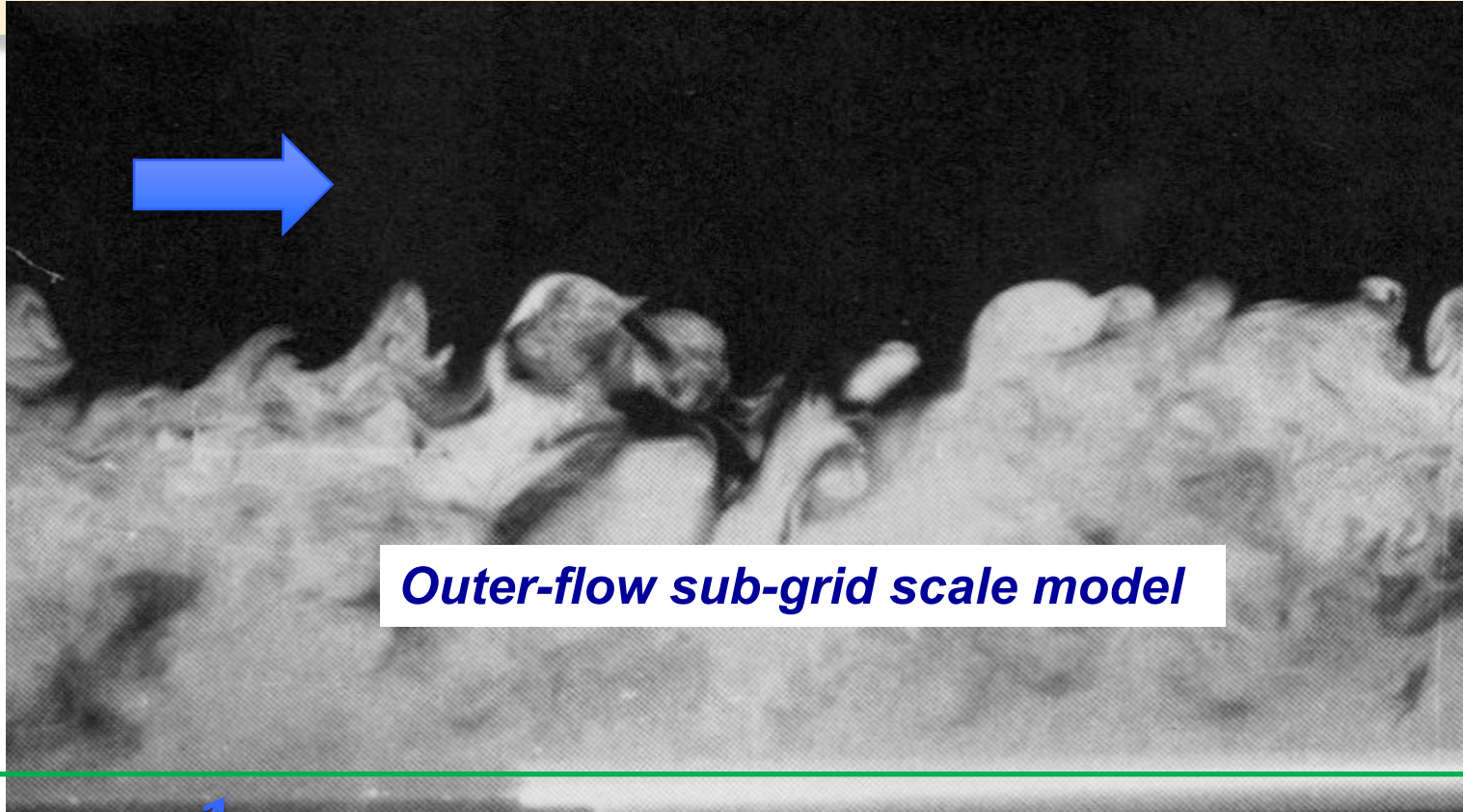
- No primary separation on bottom.
- Unsteady separation dominates the flow.



Geometric sketch	Drag/lift crisis (Symbols: hollow for experiments, filled for LES)	Instantaneous skin friction lines (red line for primary separation, blue zone for unsteady secondary separation)	Flow Mechanism	
			Interaction of separations	Turbulence effect
<b>Smooth cylinder</b> 			Yes	Yes
<b>Grooved cylinder</b> 			Yes	No
<b>Rotating cylinder</b> 			Yes	No



# *LES for wall-bounded flows*



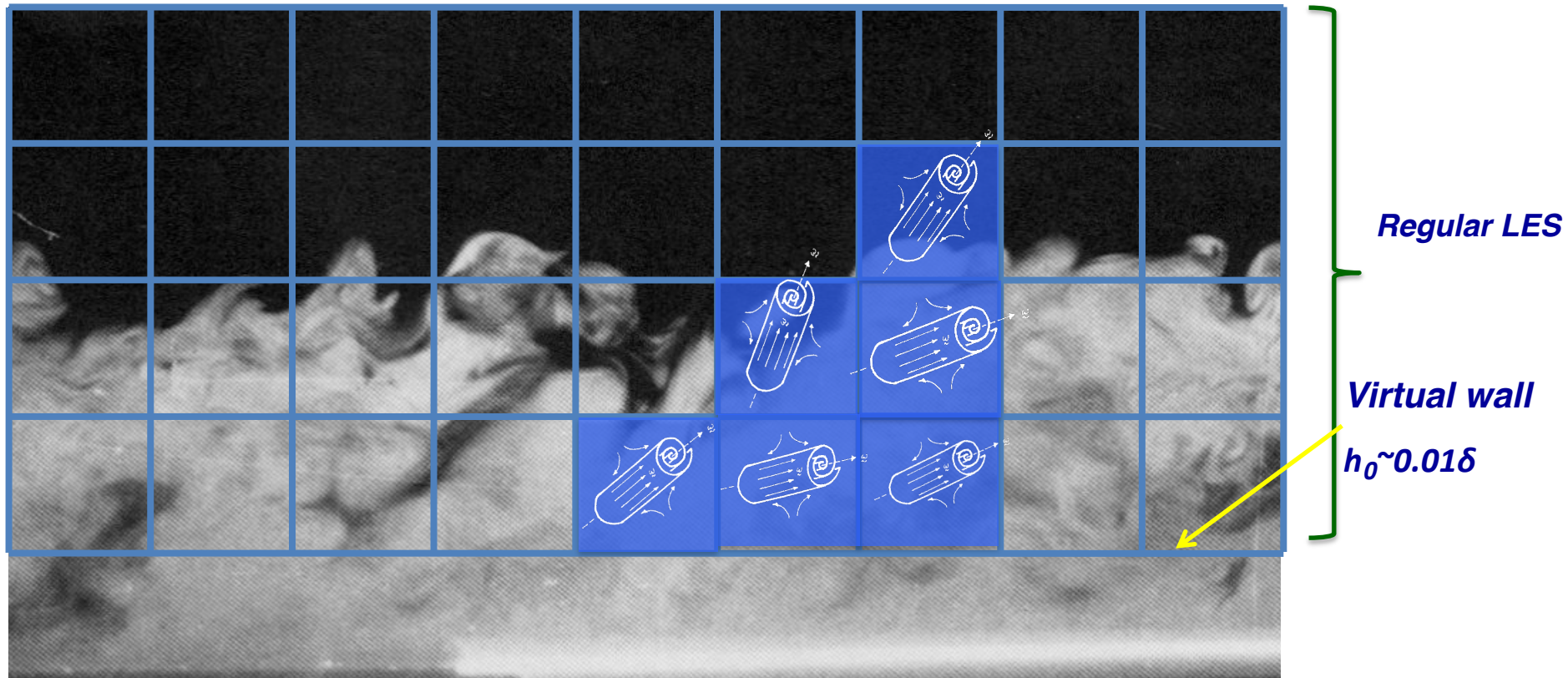
*Outer-flow sub-grid scale model*

*Wall-modeled  
region*

*True wall*

# *LES + wall model for high Re flow*

*Outer flow: LES with stretched-vortex SGS model*



# LES + wall model for high Re flow

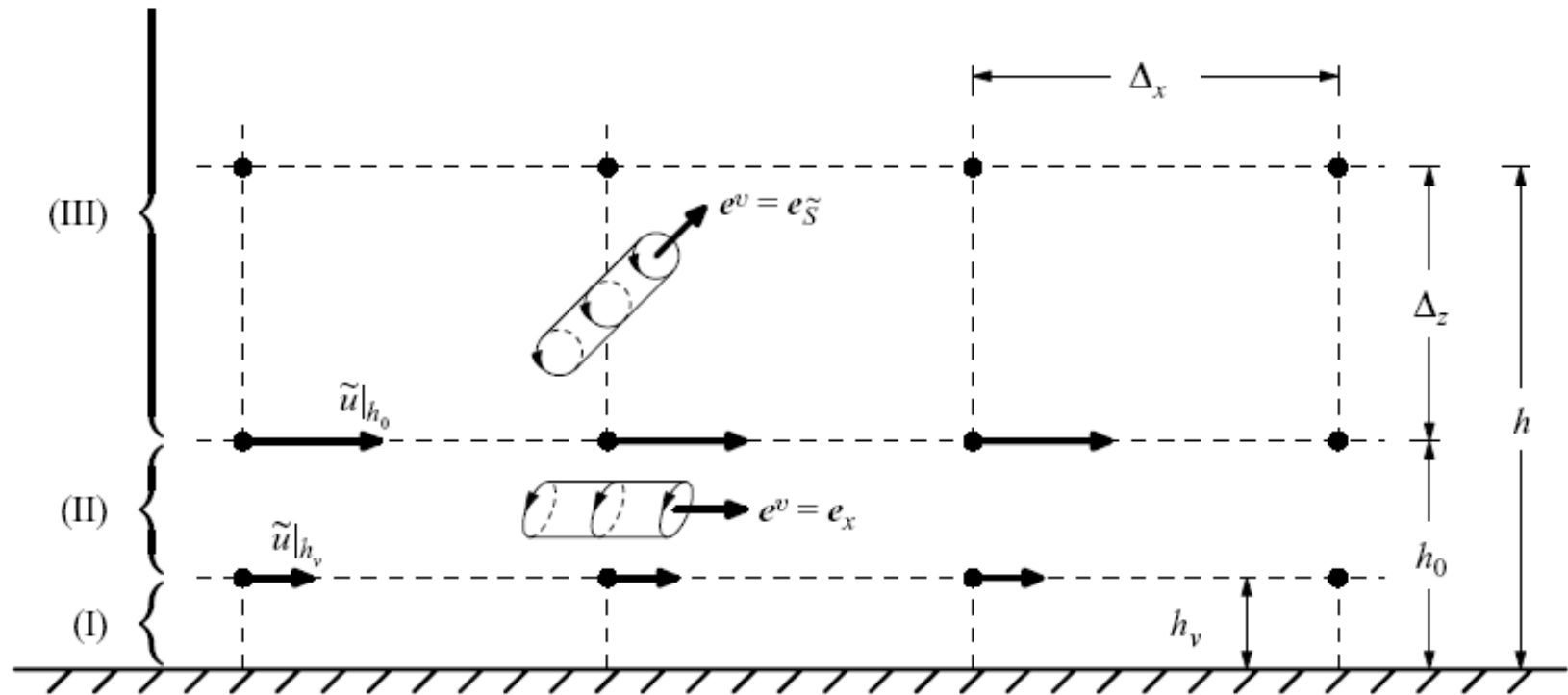
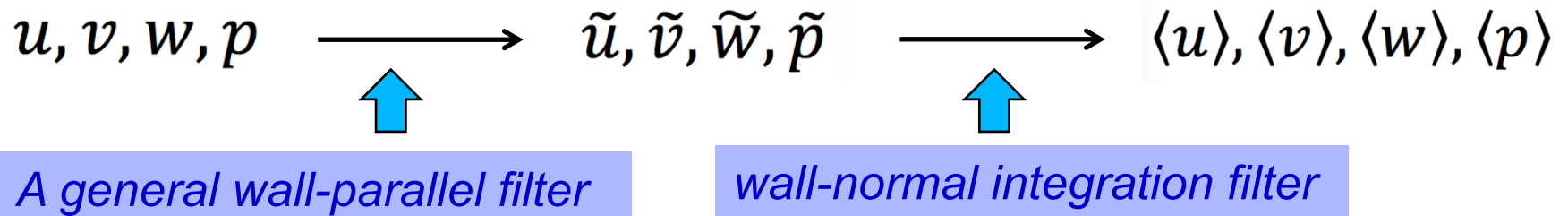


FIGURE 1. Schematic showing the near-wall set-up:  $h_0$  locates the lifted virtual wall, where boundary conditions are applied;  $h$  locates the input plane to the wall shear stress equation, (3.10);  $h_v$  locates the outer edge of the viscous sublayer;  $e^v$  is the alignment of SGS vortices in their respective regions.

# Wall Model – Essential Idea



- Inner scaling combined with wall normal integration filter

$$\frac{\tilde{q}}{u_\tau} = F(z^+), \quad z^+ \equiv \frac{z}{l^+} = \frac{zu_\tau}{\nu}. \quad \Longrightarrow \quad \frac{\partial \langle q \rangle}{\partial t} = \frac{\tilde{q}|_h}{2\eta_0} \frac{\partial \eta_0}{\partial t}.$$

- In the original model, applying the wall parallel filter

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial \tilde{u}\tilde{u}}{\partial x} + \frac{\partial \tilde{u}\tilde{v}}{\partial y} + \frac{\partial \tilde{u}\tilde{w}}{\partial z} = -\frac{\partial \tilde{p}}{\partial x} + \nu \frac{\partial^2 \tilde{u}}{\partial z^2},$$

- Main points to note
  - Classical inner scaling
  - Near wall integration approach

# Wall Model

- ODE for wall shear stress (or  $u_\tau$ ) at every wall point  $u_\tau^2 \equiv \nu \eta_0$   $\eta_0 \equiv \left. \frac{\partial \tilde{u}}{\partial z} \right|_0$ 
  - Wall-normal integration of streamwise momentum equation
  - Top-hat filter normal to the wall,  $0 < z < h$  :  $h = \Delta z > h_0$
  - Local inner-scaling reduction for unsteady term

$$\frac{\partial \eta_0}{\partial t} = \frac{2\eta_0}{\tilde{u}|_h} \left[ -\frac{\partial \tilde{u}\tilde{u}|_h}{\partial x} - \frac{\partial \tilde{u}\tilde{v}|_h}{\partial y} - \frac{\partial \tilde{u}\tilde{v}|_h}{\partial y} - \frac{\partial \tilde{p}}{\partial x} \Big|_h + \frac{\nu}{h} \left( \frac{\partial \tilde{u}}{\partial z} \Big|_h - \eta_0 \right) \right]$$

- Attached-eddy ansatz in overlap region (Townsend, 1976)
  - Hierarchy of streamwise “attached” SGS vortices whose size scales with distance from wall
  - Extended stretched-vortex SGS model with attached-eddy assumption
  - SGS model gives log relationship for slip-velocity at lifted wall position  $z = h_0$
  - “Karman constant” calculated dynamically

$$\tilde{u}|_{h_0} = u_\tau \left( \frac{1}{\mathcal{K}_1} \log \left( \frac{h_0 u_\tau / \nu}{h_\nu^+} \right) + h_\nu^+ \right)$$

$$\mathcal{K}_1 = \frac{\gamma_{II} K^{1/2}}{2 \left( -T_{xz}|_{e_{\tilde{s}}} \right)^{1/2}}$$

# Wall Model Development & Applications in LES

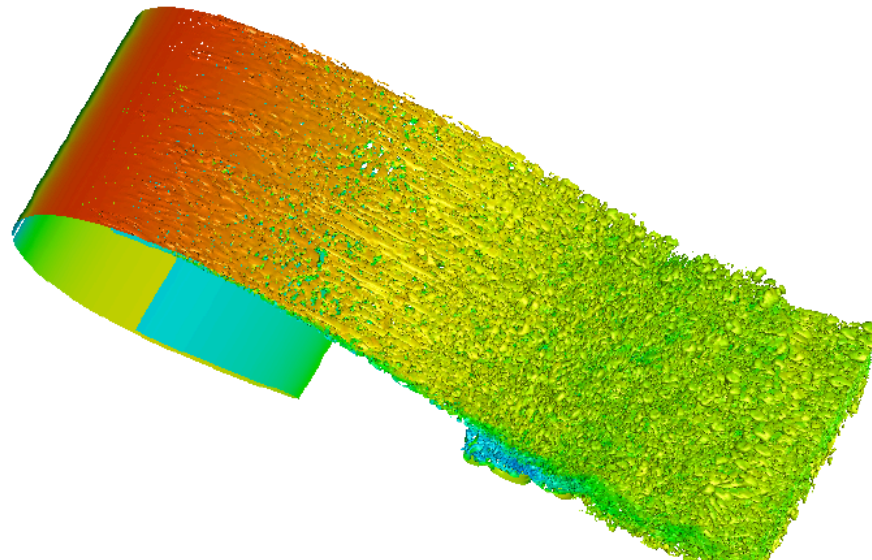
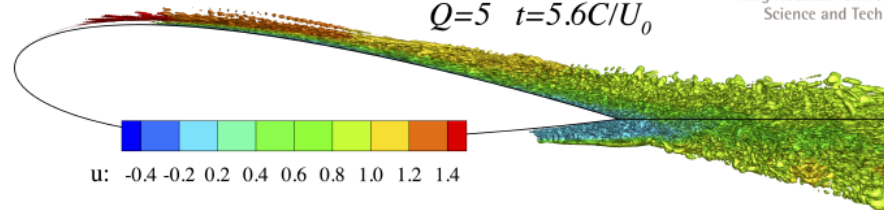
- LES of flat plate TBL: power law vs. log law
  - *Cheng & Samtaney, Phys. Fluids 2014*
- LES of separation/reattachment of flat plate TBL
  - *Cheng, Pullin, Samtaney: JFM 2015*
- LES of flow past an airfoil
  - *Gao, Cheng, Zhang, Samtaney (JFM under review)*



# Flow past Airfoils

**NACA0018,  $Re = 1.6 \times 10^5$ ,  $AoA = 6^\circ$**

$Q=5 \quad t=5.6C/U_0$

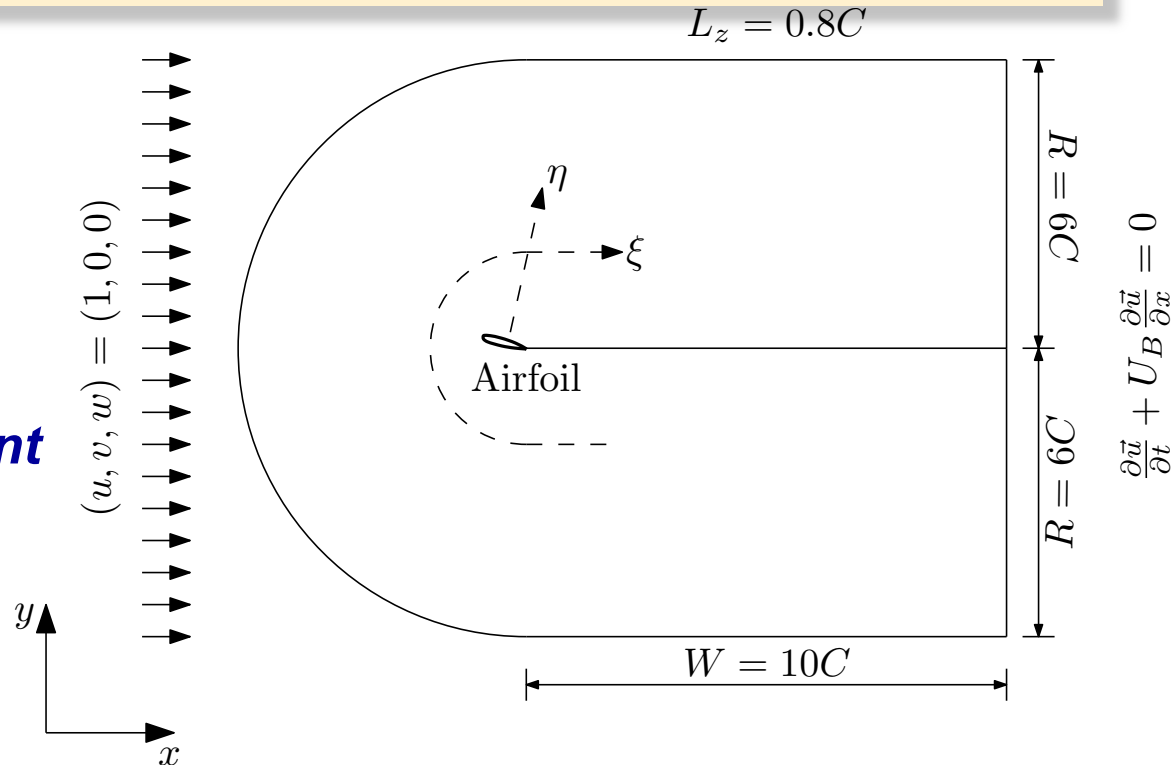


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للعلوم والتقنية  
King Abdullah University of  
Science and Technology



# Numerical Setup

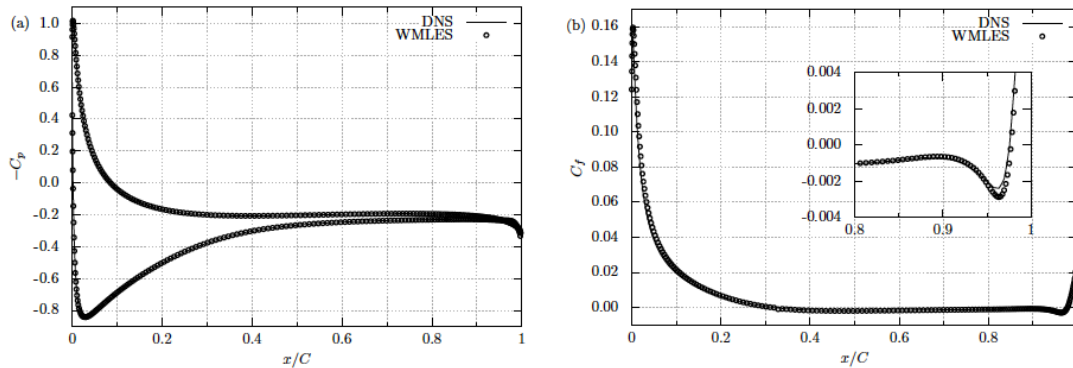
- **Comparison between DNS and WMLES at  $Re=10^4$**
- **Comparison between WMLES and experiment at  $Re=10^5$ ,  $2.1 \times 10^6$**



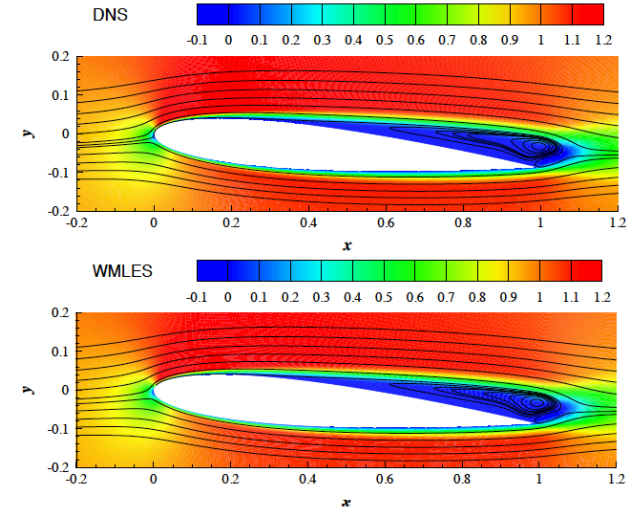
Airfoil	Method	$Re_c$	$AoA$	$N_\xi \times N_\eta \times N_z$	$\Delta \xi_{max}^+$	$\Delta \eta_{max}^+$	$\Delta z_{max}^+$
NACA0012	DNS	$10^4$	$5^\circ$	$2048 \times 256 \times 256$	7.4	0.8	8.8
NACA0012	WMLES	$10^4$	$5^\circ$	$768 \times 96 \times 64$	19.7	14.2	38.4
NACA0018	WMLES	$10^5$	$5^\circ$	$1600 \times 128 \times 128$	54.9	15.8	65.8
A-Airfoil	WMLES	$2.1 \times 10^6$	$13.3^\circ$	$3200 \times 256 \times 256$	80.1	16.4	85.4



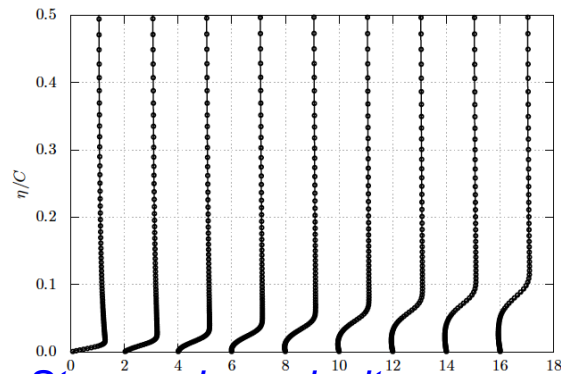
# NACA0012, $Re=10^4$ , $AOA=5$



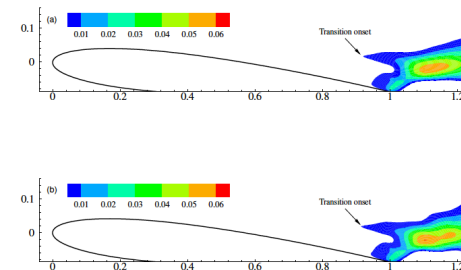
*Time and spanwise averaged pressure coefficient and skin friction coefficient*



*Time and spanwise averaged streamwise velocity component and streamlines*

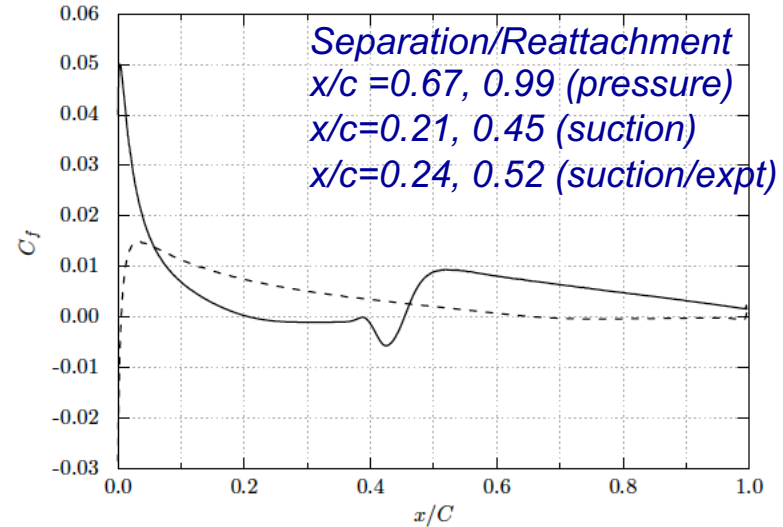
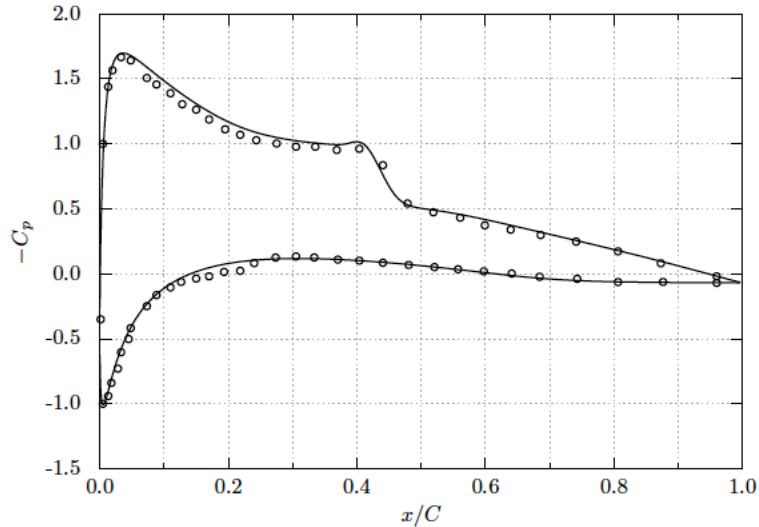


*Streamwise velocity component on suction side from  $x=0.1$  to  $0.9$ .*

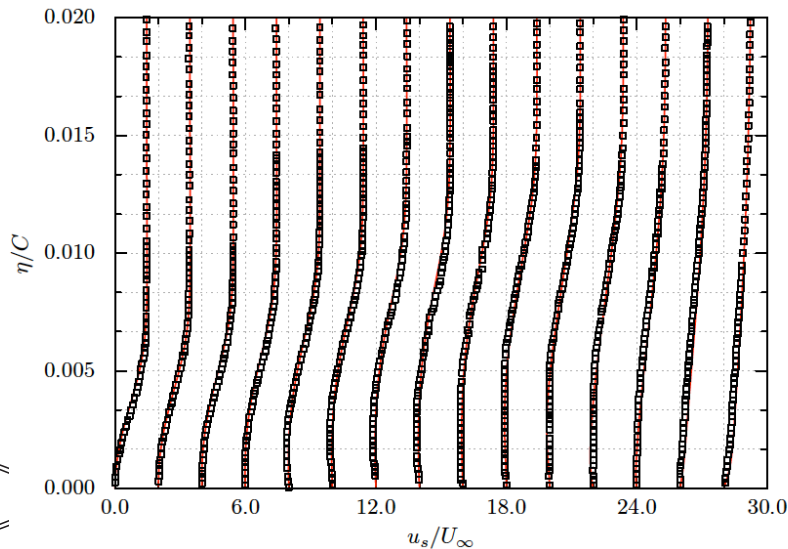


Contour plot of Reynolds stress  $(-\overline{u'v'})$ , transition criterion:  $-\overline{u'v'} = 0.001$ .

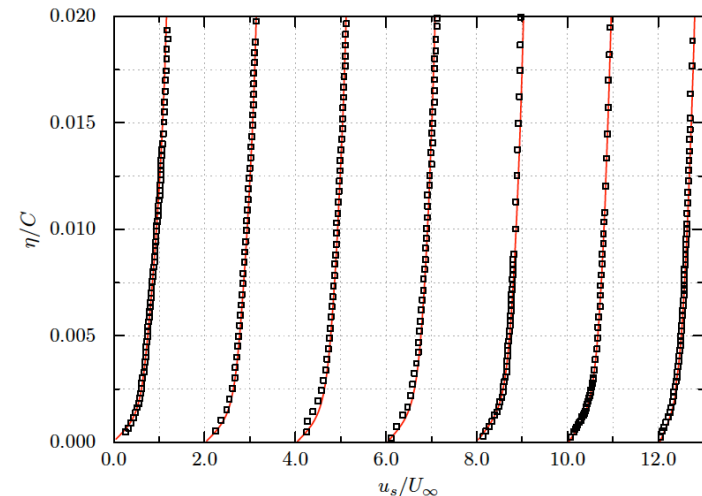
# NACA0018, $Re=10^5$ , $AOA=5$



Symbols: Expts of Kirk & Yarusevych (2017)

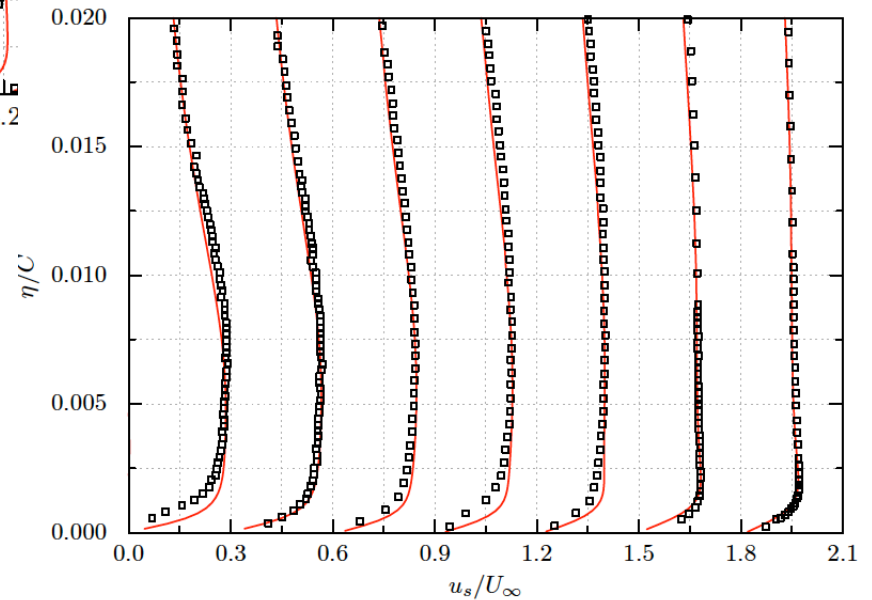
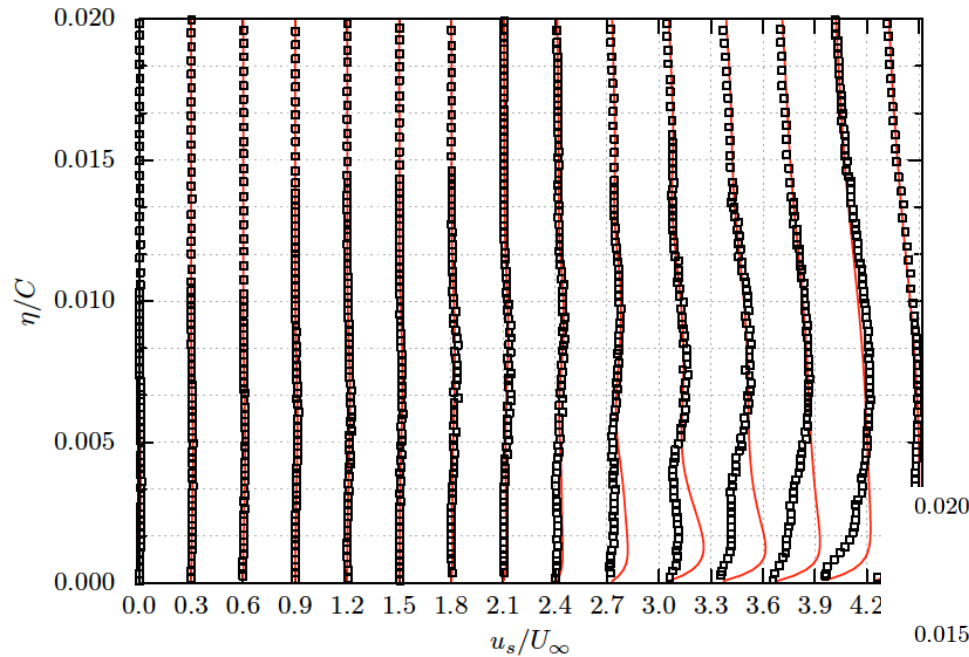


Mean velocity profiles  $x/c=0.2-0.5$

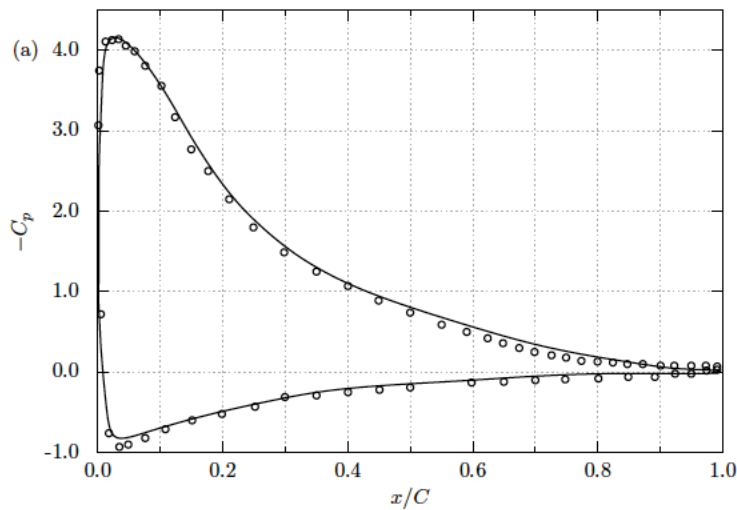


Mean velocity profiles  $x/c=0.52, 0.54, 0.6, 0.66, 0.73, 0.87$

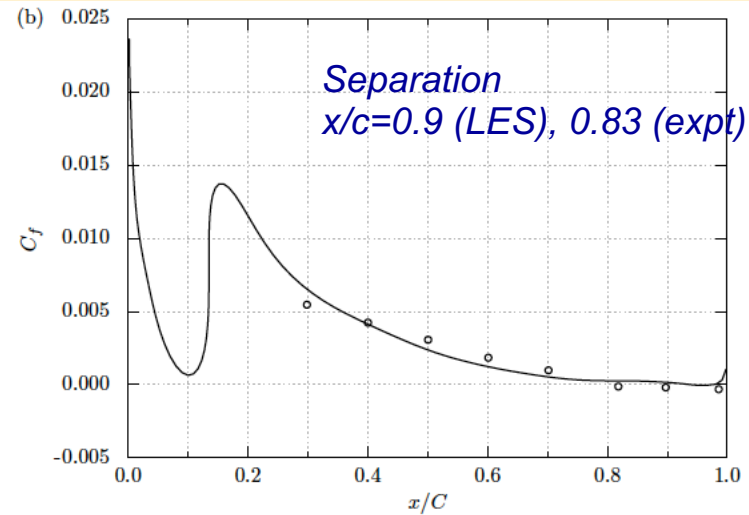
# NACA0018, $Re=10^5$ , $AOA=5$



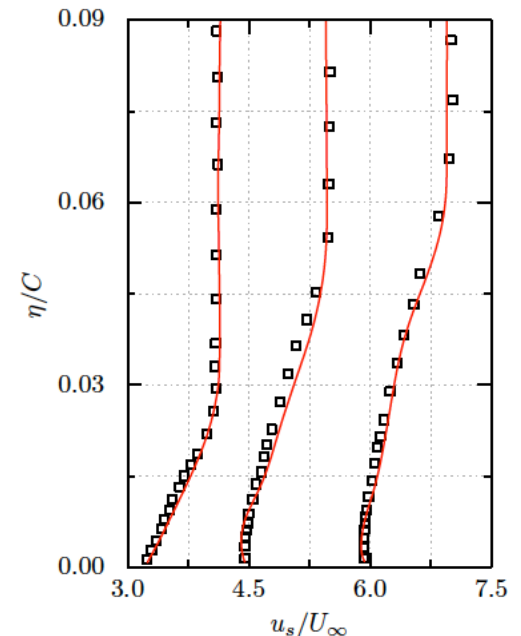
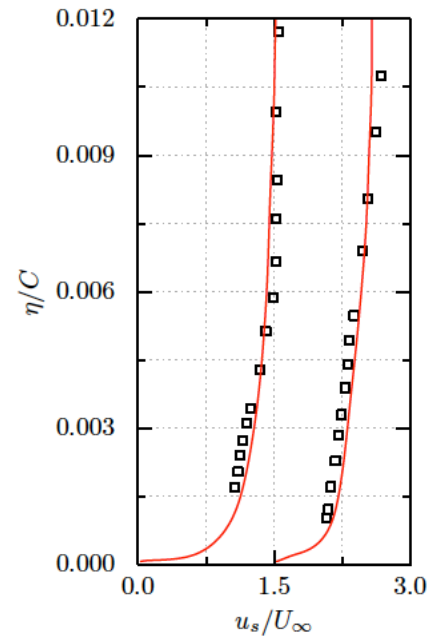
# A-airfoil, $Re=2.1 \times 10^6$ , $AOA=13.3$



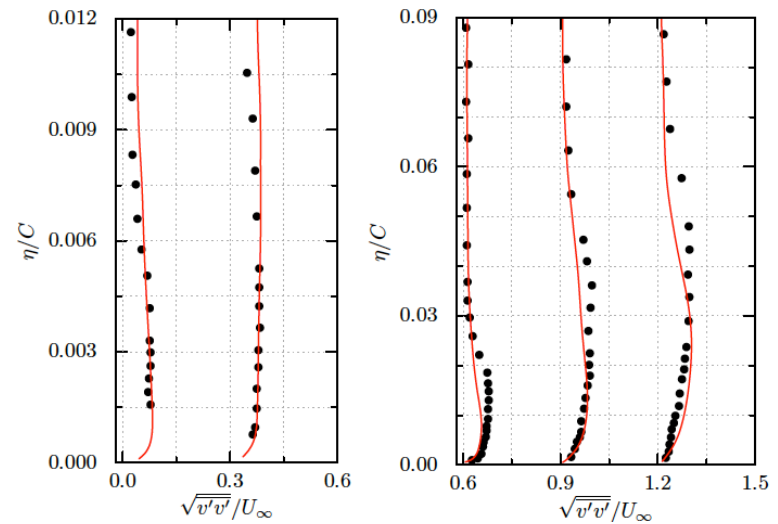
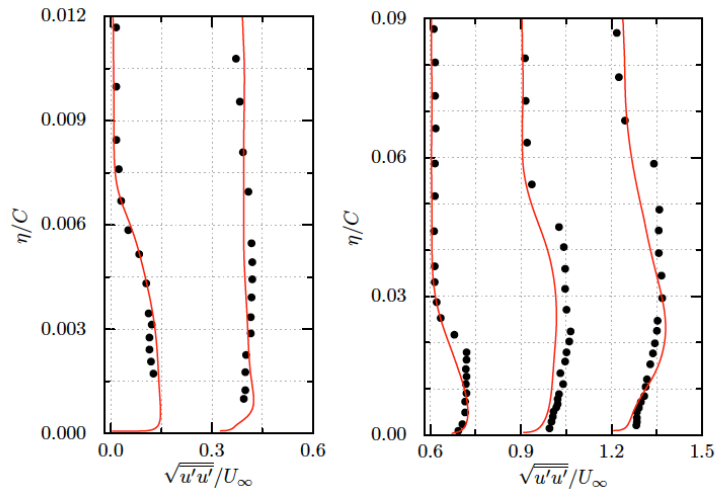
*Symbols: Expts of Mary & Sagaut (2002)*



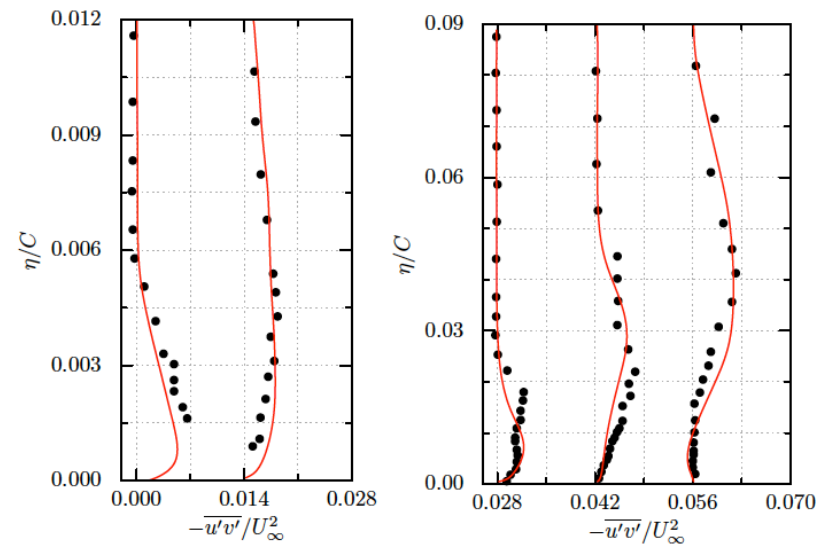
*Mean velocity profiles*



# A-airfoil, $Re=2.1 \times 10^6$ , $AOA=13.3$



*Reynolds stress*



# Conclusion

## Is unsteady separation important in other canonical flows?

- Cylinder flow:
  - Interactions between unsteady separations, turbulence transition
- Grooved cylinder flow
  - Interaction between unsteady separations, no turbulence transition on surface
- Rotating cylinder flow
  - One unsteady separation, no turbulence transition on surface
- Unsteady separation is a dominant mechanism in cylinder-type flows.
- Drag crisis observed for SC, GC, RC: turbulent transition plays a role only in SC

## Progress in flows past airfoils

- Preliminary results indicate WMLES can handle separation effectively

# Thank you

- Acknowledgement
  - *KAUST Office of Competitive Research Funds under Award Number URF/1/1394-1 and Baseline Research Funds*
  - *KAUST Supercomputing Laboratory for time on Shaheen I (IBM BGP) and Shaheen II (Cray XC-40) (10s of millions of cpu hours)*

Smooth cylinder	Grooved cylinder	Rotating cylinder
40M	35M	30M

- Questions?